

IX. EXERCISES ON VECTORS

vectorall.tex

A. DAY1

1) Evaluate

$$\sqrt{(-6)^2 + 5^2}$$

2) Evaluate

$$8 \cdot (-4) + 12 \cdot (2) - (-1) \cdot (3)$$

3) Draw the vectors $\mathbf{a} = 6\mathbf{i}$ and $\mathbf{b} = 2\mathbf{j}$. Then find and draw

a) $\mathbf{a} + \mathbf{b}$

b) $\mathbf{a} - \mathbf{b}$

4) If $\mathbf{p} = 9\mathbf{i} - 7\mathbf{j} + 5\mathbf{k}$ and $\mathbf{q} = -8\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ express

a) $\mathbf{p} + \mathbf{q}$ in terms of \mathbf{i}, \mathbf{j} , and \mathbf{k}

b) $\mathbf{p} - \mathbf{q}$ in terms of \mathbf{i}, \mathbf{j} , and \mathbf{k}

5) Sketch the position vectors

$$\mathbf{p} = 3\mathbf{i} + 4\mathbf{j}$$

and

$$\mathbf{q} = -2\mathbf{i} + 5\mathbf{j}$$

and

$$\mathbf{r} = -3\mathbf{i} - 2\mathbf{j}$$

and find the modulus of each of the vectors.

6) Draw the vectors $4\mathbf{i}$ and $5\mathbf{j}$ and, by translating the vectors so that they lie head to tail, the vector sum $4\mathbf{i} + 5\mathbf{j}$.

7) Answer the following set of problems

a) Draw an xy plane and show the vectors

$$\mathbf{p} = 2\mathbf{i} + 3\mathbf{j}$$

and

$$\mathbf{q} = 5\mathbf{i} + \mathbf{j}$$

b) Express \mathbf{p} and \mathbf{q} using column vector notation.c) By translating one of the vectors, show the sum $\mathbf{p} + \mathbf{q}$ on an xy plane.d) Express the resultant $\mathbf{p} + \mathbf{q}$ in terms of \mathbf{i} and \mathbf{j}

8) State the position vectors of the points with the coordinates

a) P(2,4,5)

b) P(-1,5,3)

c) P(-2,-1,4)

d) P(8,-4,-1)

9) Consider the points $R = (4, 7)$ and $S = (3, -3)$.

Find

a) and draw the position vector of point R .b) the vector \overrightarrow{RS} expressed in column notation

- 10) Consider the vectors $\mathbf{c} = 4\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$ and $\mathbf{d} = -6\mathbf{i} + \mathbf{j} - 7\mathbf{k}$ together with the scalar $\lambda = 3$.
Find
- $\mathbf{c} - \lambda\mathbf{d}$ expressed in terms of \mathbf{i} , \mathbf{j} and \mathbf{k}
 - the magnitude of \mathbf{c}
 - a unit vector parallel to \mathbf{c}
- 11) Point P has coordinates $(7, -4, -2)$. Point Q has coordinates $(-2, -5, -1)$
- State the position vectors of P and Q
 - Find an expression for \overrightarrow{PQ}
 - Find $|\overrightarrow{PQ}|$

B. DAY2

12) Simplify

$$2^8 \cdot 2^8$$

13) Simplify

$$3x + 2y - z + t(7x + 5y + z - (3x + 2y - z))$$

14) Evaluate

$$9 - 7(12 - 5^2)$$

15) Find the angle between the vectors

$$\mathbf{p} = -3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

and

$$\mathbf{q} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$$

16) Find $\mathbf{i} \cdot \mathbf{i}$ and $\mathbf{i} \cdot \mathbf{j}$ and $\mathbf{i} \cdot \mathbf{k}$

17) If

$$\mathbf{p} = 7\mathbf{i} + 8\mathbf{j} - \mathbf{k}$$

and

$$\mathbf{q} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

find the scalar product $\mathbf{p} \cdot \mathbf{q}$

18) If \mathbf{p} and \mathbf{q} are perpendicular, simplify $(\mathbf{p} - 2\mathbf{q}) \cdot (3\mathbf{p} + 5\mathbf{q})$

19) Points R , S , and T have coordinates $(-4, 0, -1)$, $(5, 3, -5)$ and $(2, -7, -3)$ respectively.

Find

a) the scalar product $\overrightarrow{RS} \cdot \overrightarrow{RT}$.

b) the vector product $\overrightarrow{RS} \times \overrightarrow{RT}$.

c) the angle between the vectors \overrightarrow{RS} and \overrightarrow{RT} .

20) Find a vector which is perpendicular to both of the vectors

$$\mathbf{c} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

and

$$\mathbf{d} = \mathbf{i} - 4\mathbf{j} - 6\mathbf{k}.$$

21) Points P , Q , and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

22) Evaluate the vector product $\mathbf{p} \times \mathbf{q}$ if

$$\mathbf{p} = 3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$$

and

$$\mathbf{q} = 7\mathbf{i} + 4\mathbf{j} - 8\mathbf{k}$$

23) Find the vector product of

$$\mathbf{p} = -2\mathbf{i} - 3\mathbf{j}$$

and

$$\mathbf{q} = 4\mathbf{i} + 7\mathbf{j}$$

24) If

$$\mathbf{p} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$$

and

$$\mathbf{q} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k},$$

find $\mathbf{p} \times \mathbf{q}$ and $\mathbf{q} \times \mathbf{p}$

25) Find a vector which is perpendicular to both of the vectors

$$\mathbf{p} = \mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$$

and

$$\mathbf{q} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}.$$

Hence find a unit vector which is perpendicular to both \mathbf{p} and \mathbf{q} .

26) For the vectors

$$\mathbf{p} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

$$\mathbf{q} = \mathbf{i} - 2\mathbf{j} + \mathbf{k},$$

and

$$\mathbf{r} = 3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k},$$

evaluate both $\mathbf{p} \times (\mathbf{q} \times \mathbf{r})$ and $(\mathbf{p} \times \mathbf{q}) \times \mathbf{r}$.