EEEN19680 Supplementary Maths (nee Engineering Mathematics)

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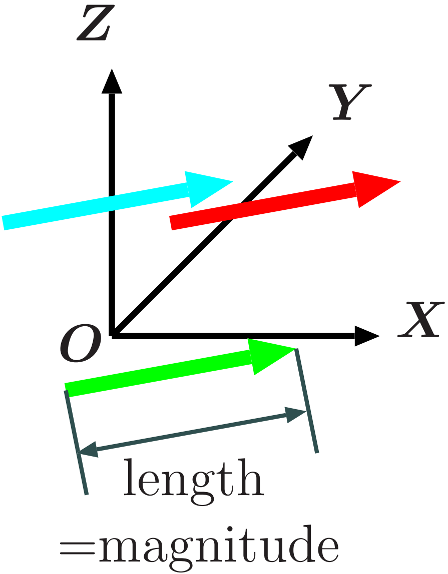
vectorDAY1, vectorDAY2

Definition of a vector

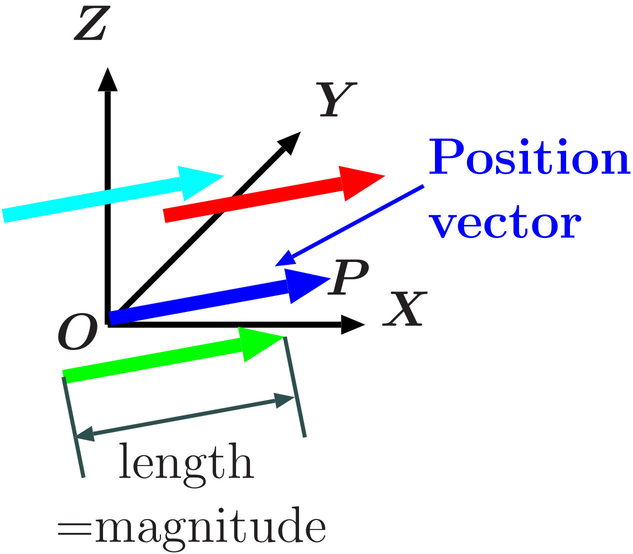


- **direction**
- **length=magnitude**

All these 3 vectors are identical

Definition of a position vector

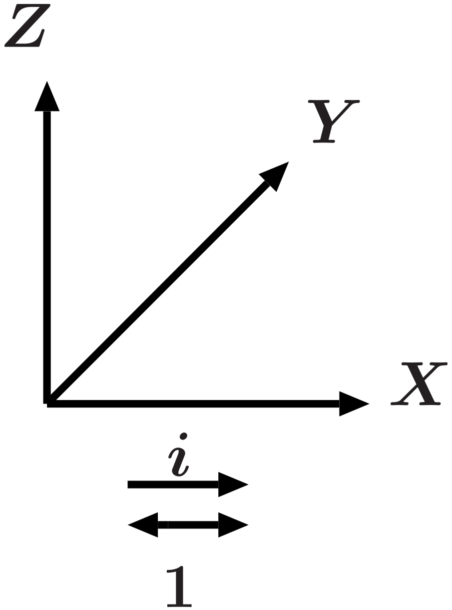
Definition of a position vector



Position vector

- Starts from origin O
- Shows the location of a point P

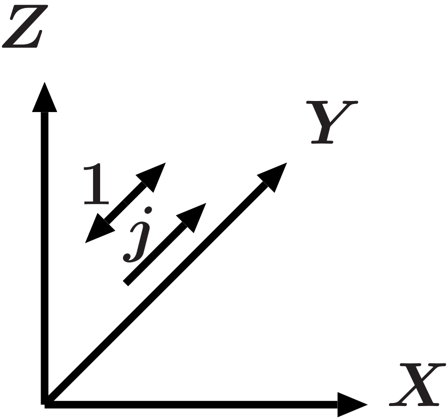
A unit vector in x direction



Column vector notation

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

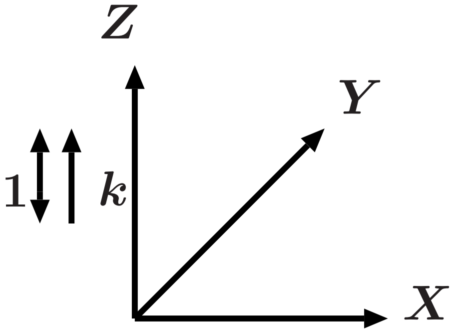
A unit vector in *y* direction



Column vector notation

$$\boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

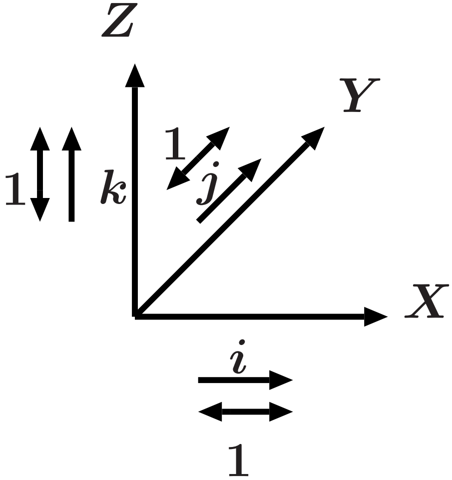
A unit vector in *z* direction



Column vector notation

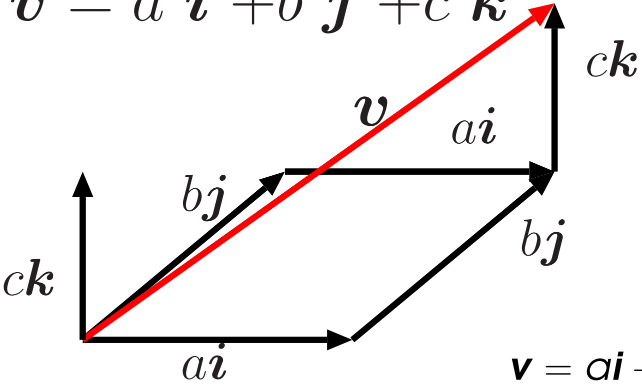
$$\boldsymbol{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Unit vectors ***i*** , ***j*** , ***k***



Expression of a vector *v* with unit vectors *i* , *j* , *k*

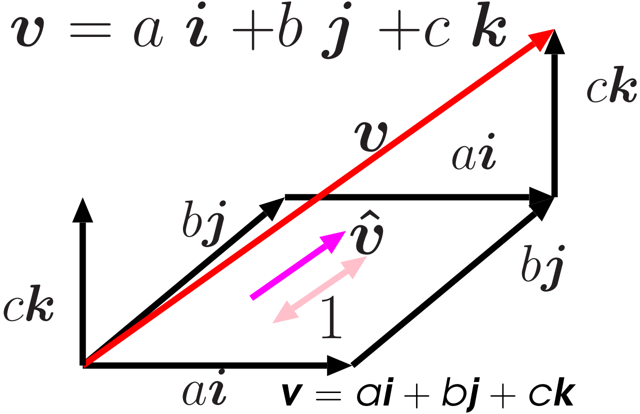
$$\boldsymbol{v} = a\,\boldsymbol{i} + b\,\boldsymbol{j} + c\,\boldsymbol{k}$$



$$\boldsymbol{v} = a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$$

$$= a\begin{pmatrix}1\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\0\end{pmatrix} + c\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}a\\b\\c\end{pmatrix}$$

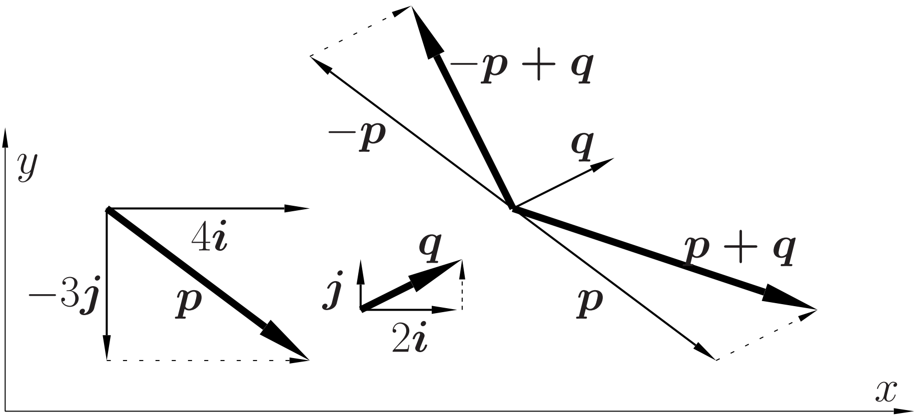
Magnitude (=modulus) of a vector *v* and the unit vector of *v*



Magnitude $|\boldsymbol{v}|$ is $\sqrt{a^2+b^2+c^2}$

The unit vector of *v*, *i.e.*, $\hat{\boldsymbol{v}}$ is $\dfrac{\boldsymbol{v}}{|\boldsymbol{v}|}$

Graphical vector addition



Numerical vector addition

If

$$\boldsymbol{a} = a_1\boldsymbol{i} + a_2\boldsymbol{j} + a_3\boldsymbol{k} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

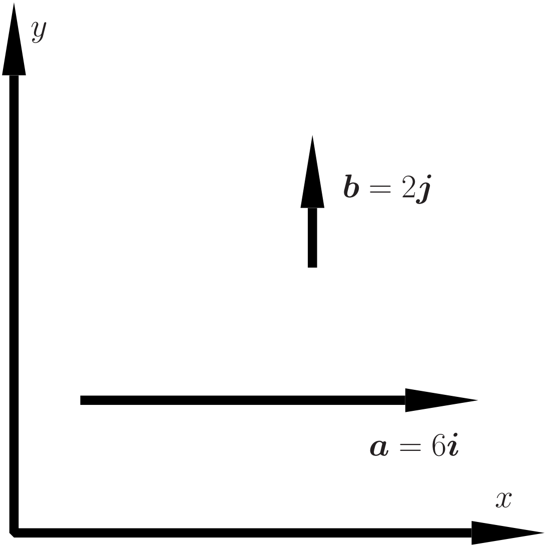
$$\boldsymbol{b} = b_1\boldsymbol{i} + b_2\boldsymbol{j} + b_3\boldsymbol{k} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

then

$$\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{pmatrix}$$

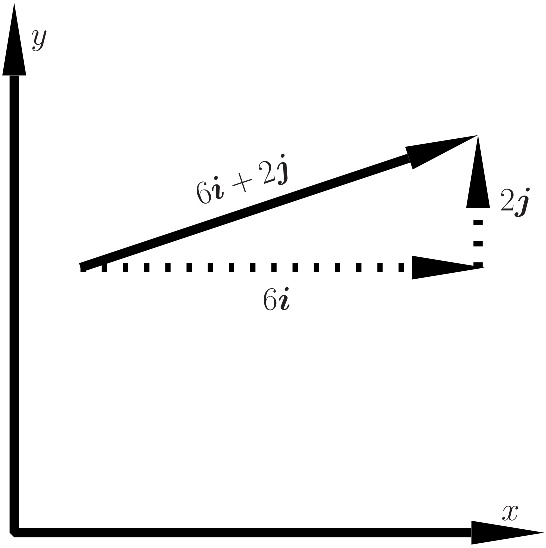
Draw the vectors $\boldsymbol{a} = 6\boldsymbol{i}$ and $\boldsymbol{b} = 2\boldsymbol{j}$.

Draw the vectors $\boldsymbol{a} = 6\boldsymbol{i}$ and $\boldsymbol{b} = 2\boldsymbol{j}$.



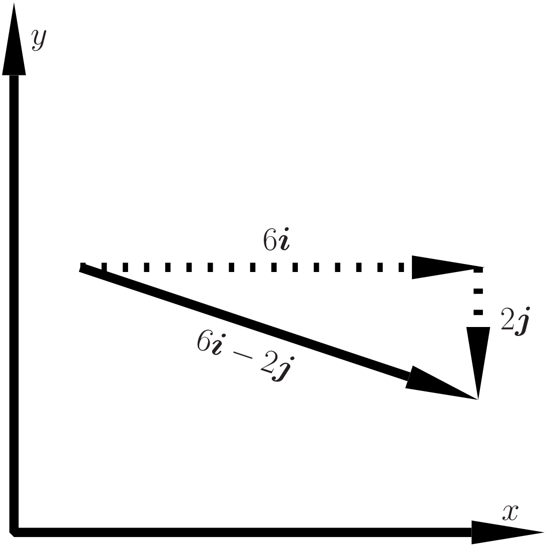
Draw $\boldsymbol{a} + \boldsymbol{b}$ when $\boldsymbol{a} = 6\boldsymbol{i}$ and $\boldsymbol{b} = 2\boldsymbol{j}$.

Draw $\boldsymbol{a} + \boldsymbol{b}$ when $\boldsymbol{a} = 6\boldsymbol{i}$ and $\boldsymbol{b} = 2\boldsymbol{j}$.



Draw $\boldsymbol{a} - \boldsymbol{b}$ when $\boldsymbol{a} = 6\boldsymbol{i}$ and $\boldsymbol{b} = 2\boldsymbol{j}$.

Draw $\boldsymbol{a} - \boldsymbol{b}$ when $\boldsymbol{a} = 6\boldsymbol{i}$ and $\boldsymbol{b} = 2\boldsymbol{j}$.



Express $\boldsymbol{p}+\boldsymbol{q}$ in terms of $\boldsymbol{i}$,$\boldsymbol{j}$, and $\boldsymbol{k}$ when $\boldsymbol{p}=9\boldsymbol{i}-7\boldsymbol{j}+5\boldsymbol{k}$ and $\boldsymbol{q}=-8\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k}$

$$\boldsymbol{p}+\boldsymbol{q}=$$

Express $\boldsymbol{p} + \boldsymbol{q}$ in terms of $\boldsymbol{i}$,$\boldsymbol{j}$, and $\boldsymbol{k}$ when $\boldsymbol{p} = 9\boldsymbol{i} - 7\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{q} = -8\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$

$$\boldsymbol{p} + \boldsymbol{q} = \begin{pmatrix} 9 \\ -7 \\ 5 \end{pmatrix} + \begin{pmatrix} -8 \\ 3 \\ -2 \end{pmatrix}$$

$$=$$

Express $\boldsymbol{p}+\boldsymbol{q}$ in terms of $\boldsymbol{i}$,$\boldsymbol{j}$, and $\boldsymbol{k}$ when $\boldsymbol{p} = 9\boldsymbol{i} - 7\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{q} = -8\boldsymbol{i} + 3\boldsymbol{j} - 2\boldsymbol{k}$

$$\boldsymbol{p}+\boldsymbol{q} = \begin{pmatrix} 9 \\ -7 \\ 5 \end{pmatrix} + \begin{pmatrix} -8 \\ 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 9-8 \\ -7+3 \\ 5-2 \end{pmatrix} =$$

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$$= \begin{pmatrix} 9-8 \\ -7+3 \\ 5-2 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}$$

Therefore $\boldsymbol{p} + \boldsymbol{q} =$

Express $\boldsymbol{p}+\boldsymbol{q}$ in terms of $\boldsymbol{i}$,$\boldsymbol{j}$, and $\boldsymbol{k}$ when $\boldsymbol{p}=9\boldsymbol{i}-7\boldsymbol{j}+5\boldsymbol{k}$ and $\boldsymbol{q}=-8\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k}$

$$\boldsymbol{p}+\boldsymbol{q}=\begin{pmatrix}9\\-7\\5\end{pmatrix}+\begin{pmatrix}-8\\3\\-2\end{pmatrix}$$

$$=\begin{pmatrix}9-8\\-7+3\\5-2\end{pmatrix}=\begin{pmatrix}1\\-4\\3\end{pmatrix}$$

Therefore $\boldsymbol{p}+\boldsymbol{q}=\boldsymbol{i}-4\boldsymbol{j}+3\boldsymbol{k}$

State the position vectors of the point with the coordinates of P(2,4,5)

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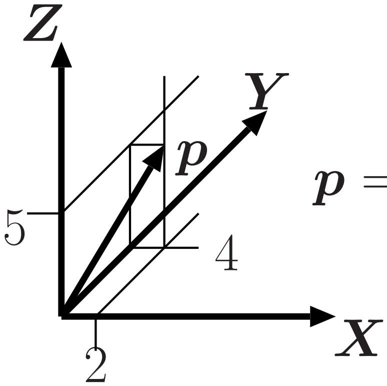
$$\boldsymbol{p} = \overrightarrow{OP} =$$

State the position vectors of the point with the coordinates of P(2,4,5)

$$\boldsymbol{p} = \overrightarrow{OP} = 2\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$$



$$\boldsymbol{p} = \vec{OP} = 2\ \boldsymbol{i} + 4\ \boldsymbol{j} + 5\ \boldsymbol{k}$$

Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$

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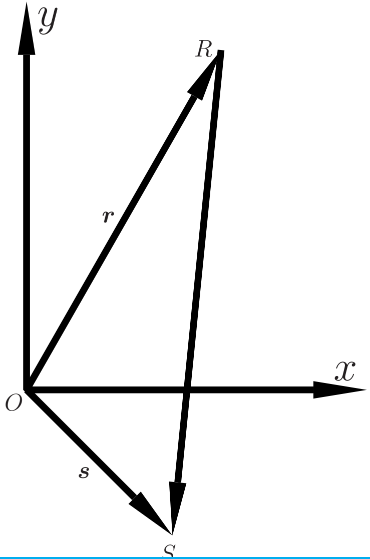
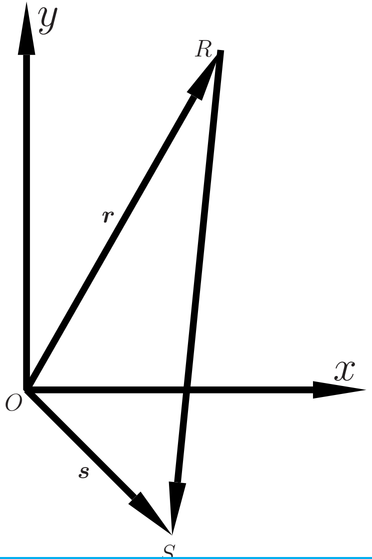


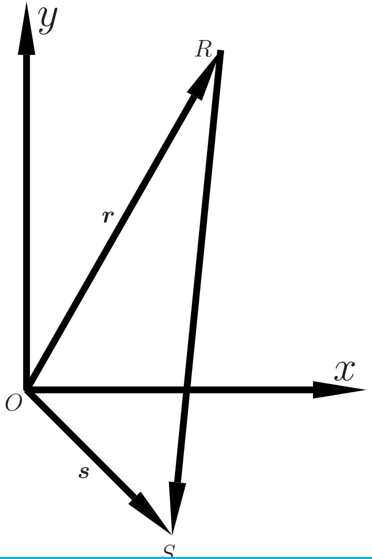
$\overrightarrow{RS} =$

Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$



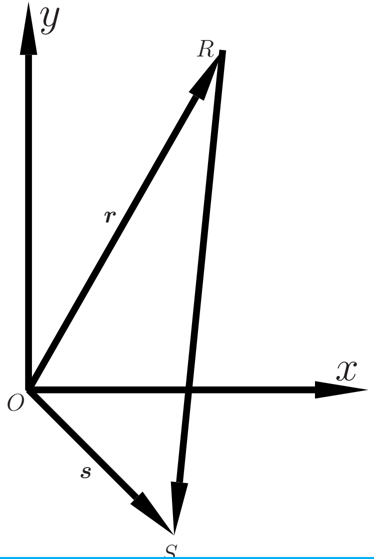
$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$
$$=$$

Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$



$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$
$$= -\overrightarrow{OR} + \overrightarrow{OS}$$
$$=$$

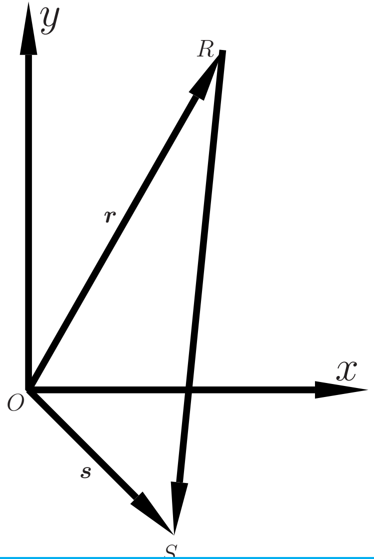
Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$



$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$
$$= -\overrightarrow{OR} + \overrightarrow{OS}$$
$$= -\boldsymbol{r} + \boldsymbol{s}$$

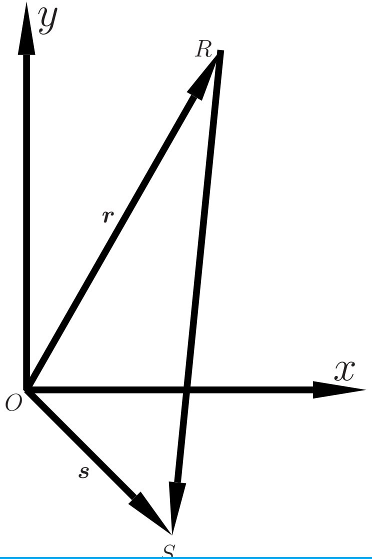
$$=$$

Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$



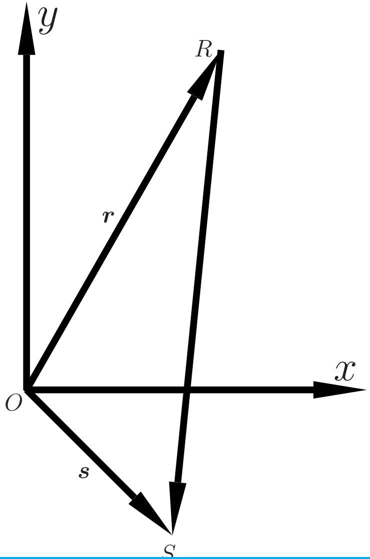
$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$
$$= -\overrightarrow{OR} + \overrightarrow{OS}$$
$$= -\boldsymbol{r} + \boldsymbol{s}$$
$$= -\begin{pmatrix} 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$
$$=$$

Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$



$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$
$$= -\overrightarrow{OR} + \overrightarrow{OS}$$
$$= -\boldsymbol{r} + \boldsymbol{s}$$
$$= -\begin{pmatrix} 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} -4 + 3 \\ -7 - 3 \end{pmatrix}$$
$$=$$

Express the vector $\overrightarrow{RS}$ in column notation when $R = (4, 7)$ and $S = (3, -3)$



$$\overrightarrow{RS} = \overrightarrow{RO} + \overrightarrow{OS}$$
$$= -\overrightarrow{OR} + \overrightarrow{OS}$$
$$= -\boldsymbol{r} + \boldsymbol{s}$$
$$= -\begin{pmatrix} 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$
$$= \begin{pmatrix} -4+3 \\ -7-3 \end{pmatrix}$$
$$= \begin{pmatrix} -1 \\ -10 \end{pmatrix}$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find $\boldsymbol{c} - \lambda\boldsymbol{d}$ expressed in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find $\boldsymbol{c} - \lambda\boldsymbol{d}$ expressed in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$

$$\boldsymbol{c} - \lambda\boldsymbol{d} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} -$$

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$$\boldsymbol{c} - \lambda\boldsymbol{d} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - 3 \cdot \begin{pmatrix} -6 \\ 1 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix}$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find $\boldsymbol{c} - \lambda\boldsymbol{d}$ expressed in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$

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$$\boldsymbol{c} - \lambda\boldsymbol{d} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - 3 \cdot \begin{pmatrix} -6 \\ 1 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - \begin{pmatrix} 3 \cdot (-6) \\ 3 \cdot 1 \\ 3 \cdot (-7) \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - \begin{pmatrix} -18 \\ 3 \\ -21 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ \\ \end{pmatrix}$$

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$$= \left(\begin{matrix} 4 - (-18) \\ -5 \\ \end{matrix} \right.$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find $\boldsymbol{c} - \lambda\boldsymbol{d}$ expressed in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$

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Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find $\boldsymbol{c} - \lambda\boldsymbol{d}$ expressed in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$

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$$= \begin{pmatrix} 4 - (-18) \\ -5 - 3 \\ 10 - (-21) \end{pmatrix} = \begin{pmatrix} 22 \\ -8 \\ 31 \end{pmatrix}$$

$$=$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find $\boldsymbol{c} - \lambda\boldsymbol{d}$ expressed in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$

$$\boldsymbol{c} - \lambda\boldsymbol{d} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - 3 \cdot \begin{pmatrix} -6 \\ 1 \\ -7 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - \begin{pmatrix} 3 \cdot (-6) \\ 3 \cdot 1 \\ 3 \cdot (-7) \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \\ 10 \end{pmatrix} - \begin{pmatrix} -18 \\ 3 \\ -21 \end{pmatrix}$$

$$= \begin{pmatrix} 4 - (-18) \\ -5 - 3 \\ 10 - (-21) \end{pmatrix} = \begin{pmatrix} 22 \\ -8 \\ 31 \end{pmatrix}$$

$$= 22\boldsymbol{i} - 8\boldsymbol{j} + 31\boldsymbol{k}$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$.
Find the magnitude of $\boldsymbol{c}$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find the magnitude of $\boldsymbol{c}$

$$|\boldsymbol{c}| =$$

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$$|\boldsymbol{c}| = \sqrt{4^2 + (-5)^2 + 10^2}$$

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$$
\begin{aligned}
|\boldsymbol{c}| &= \sqrt{4^2 + (-5)^2 + 10^2} \\
&= \sqrt{16 + 25 + 100}
\end{aligned}
$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$.
Find the magnitude of $\boldsymbol{c}$

$$
\begin{aligned}
|\boldsymbol{c}| &= \sqrt{4^2 + (-5)^2 + 10^2} \\
&= \sqrt{16 + 25 + 100} \\
&= \sqrt{141}
\end{aligned}
$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find a unit vector parallel to $\boldsymbol{c}$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find a unit vector parallel to $\boldsymbol{c}$

$$\hat{\boldsymbol{n}} =$$

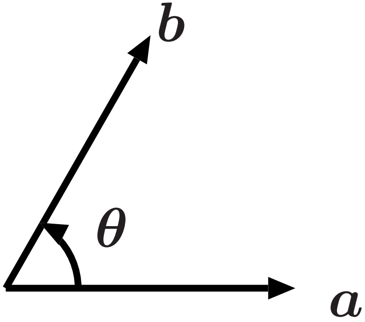
Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find a unit vector parallel to $\boldsymbol{c}$

$$\hat{\boldsymbol{n}} = \frac{\boldsymbol{c}}{|\boldsymbol{c}|} =$$

Consider the vectors $\boldsymbol{c} = 4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k}$ and $\boldsymbol{d} = -6\boldsymbol{i} + \boldsymbol{j} - 7\boldsymbol{k}$ together with the scalar $\lambda = 3$. Find a unit vector parallel to $\boldsymbol{c}$

$$\hat{\boldsymbol{n}} = \frac{\boldsymbol{c}}{|\boldsymbol{c}|} = \frac{1}{\sqrt{141}}(4\boldsymbol{i} - 5\boldsymbol{j} + 10\boldsymbol{k})$$

Scalar product



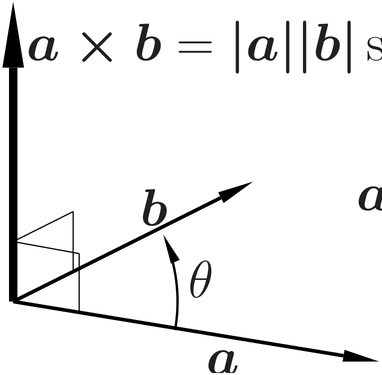
two vectors subtend an angle θ

The scalar product:

$$\begin{aligned} & \boldsymbol{a} \cdot \boldsymbol{b} \\ = {} & a_1 \cdot b_1 + a_2 \cdot b_2 + a_3 \cdot b_3 \\ = {} & |\boldsymbol{a}||\boldsymbol{b}| \cos \theta \end{aligned}$$

produces a scalar.
See the proof in the keynote

Visual vector product



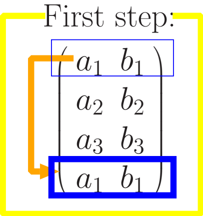
$$\boldsymbol{a} \times \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \sin\theta \hat{\boldsymbol{n}}$$ ← vector product

$$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$$

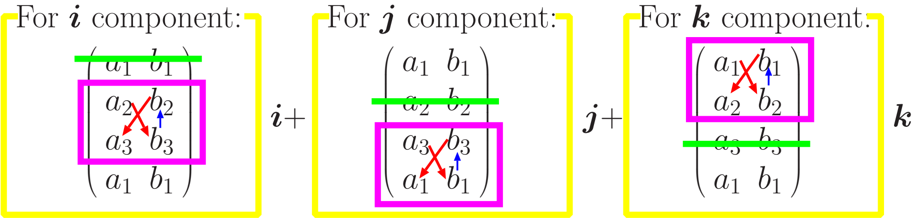
↑ scalar product

Numerical vector product of $\boldsymbol{a} \times \boldsymbol{b}$

$$(\boldsymbol{a} \ \ \boldsymbol{b}) = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \end{pmatrix}$$

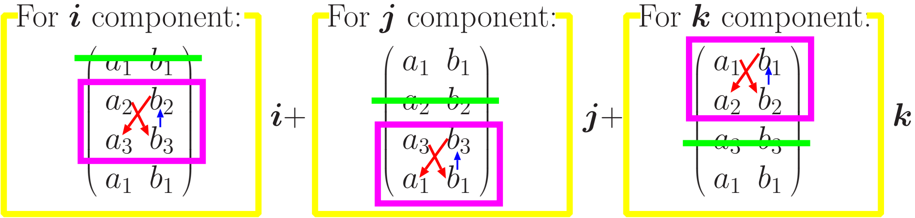


Second step:



Numerical vector product of $\boldsymbol{a} \times \boldsymbol{b}$

Second step:



$$= (a_2 b_3 - a_3 b_2)\boldsymbol{i} + (a_3 b_1 - a_1 b_3)\boldsymbol{j} + (a_1 b_2 - a_2 b_1)\boldsymbol{k}$$

$$= |\boldsymbol{a}||\boldsymbol{b}| \sin\theta \hat{\boldsymbol{n}}$$

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

Which knowledge should we use ?

$\Downarrow$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

Which knowledge should we use ?

$\Downarrow$

Scalar product!

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

Which knowledge should we use ?

$\Downarrow$

Scalar product!

What is needed to use scalar product ?

$\Downarrow$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

Which knowledge should we use ?

$$\Downarrow$$

Scalar product!

What is needed to use scalar product ?

$$\Downarrow$$

Find $\boldsymbol{p} \cdot \boldsymbol{q}$, $|\boldsymbol{p}|$, $|\boldsymbol{q}|$

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} =$$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$=$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) =$$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) = -6 - 3 + 2 = -7$$

$$|\boldsymbol{p}| =$$

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) = -6 - 3 + 2 = -7$$

$$|\boldsymbol{p}| = \sqrt{(-3)^2 + (-1)^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\boldsymbol{q}| =$$

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) = -6 - 3 + 2 = -7$$

$$|\boldsymbol{p}| = \sqrt{(-3)^2 + (-1)^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\boldsymbol{q}| = \sqrt{(2)^2 + (3)^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$\boldsymbol{p} \cdot \boldsymbol{q} = |\boldsymbol{p}| \cdot |\boldsymbol{q}| \cdot \cos\theta \therefore$$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) = -6 - 3 + 2 = -7$$

$$|\boldsymbol{p}| = \sqrt{(-3)^2 + (-1)^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\boldsymbol{q}| = \sqrt{(2)^2 + (3)^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$\boldsymbol{p} \cdot \boldsymbol{q} = |\boldsymbol{p}| \cdot |\boldsymbol{q}| \cdot \cos\theta \therefore -7 = \sqrt{14} \cdot \sqrt{14} \cos\theta$$

$\therefore$

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) = -6 - 3 + 2 = -7$$

$$|\boldsymbol{p}| = \sqrt{(-3)^2 + (-1)^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\boldsymbol{q}| = \sqrt{(2)^2 + (3)^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$\boldsymbol{p} \cdot \boldsymbol{q} = |\boldsymbol{p}| \cdot |\boldsymbol{q}| \cdot \cos\theta \therefore -7 = \sqrt{14} \cdot \sqrt{14} \cos\theta$$

$$\therefore -7 = 14 \cos\theta; \therefore$$

Find the angle between the vectors
$\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

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$$|\boldsymbol{p}| = \sqrt{(-3)^2 + (-1)^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

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$$\boldsymbol{p} \cdot \boldsymbol{q} = |\boldsymbol{p}| \cdot |\boldsymbol{q}| \cdot \cos\theta \therefore -7 = \sqrt{14} \cdot \sqrt{14} \cos\theta$$

$$\therefore -7 = 14 \cos\theta; \therefore \frac{-1}{2} = \cos\theta; \therefore \theta =$$

Find the angle between the vectors $\boldsymbol{p} = -3\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{q} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$

$$\boldsymbol{p} \cdot \boldsymbol{q} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$$

$$= (-3) \cdot 2 + (-1) \cdot (3) + (2) \cdot (1) = -6 - 3 + 2 = -7$$

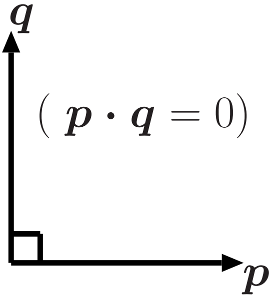
$$|\boldsymbol{p}| = \sqrt{(-3)^2 + (-1)^2 + 2^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

$$|\boldsymbol{q}| = \sqrt{(2)^2 + (3)^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$$

$$\boldsymbol{p} \cdot \boldsymbol{q} = |\boldsymbol{p}| \cdot |\boldsymbol{q}| \cdot \cos\theta \therefore -7 = \sqrt{14} \cdot \sqrt{14} \cos\theta$$

$$\therefore -7 = 14 \cos\theta; \therefore \frac{-1}{2} = \cos\theta; \therefore \theta = 120^o = \frac{2\pi}{3}$$

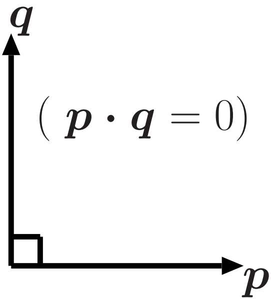
Simplify $(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q})$ when $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular



$$(\boldsymbol{p} \cdot \boldsymbol{q} = 0) \quad =$$

$$(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q})$$

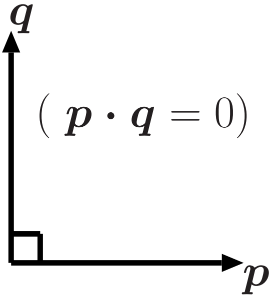
Simplify $(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q})$ when $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular



$(\, \boldsymbol{p} \bullet \boldsymbol{q} = 0)$

$$
\begin{aligned}
&(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q}) \\
&= 3\boldsymbol{p} \cdot \boldsymbol{p} + 5\boldsymbol{q} \cdot \boldsymbol{p} - 2\boldsymbol{q} \cdot 3\boldsymbol{p} - 2\boldsymbol{q} \cdot 5\boldsymbol{q} \\
&=
\end{aligned}
$$

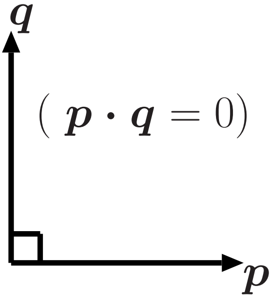
Simplify $(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q})$ when $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular



$$(\boldsymbol{p} \cdot \boldsymbol{q} = 0)$$

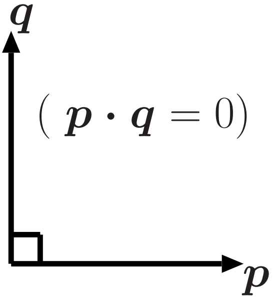
$$\begin{aligned} & (\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q}) \\ = & \; 3\boldsymbol{p} \cdot \boldsymbol{p} + 5\boldsymbol{q} \cdot \boldsymbol{p} - 2\boldsymbol{q} \cdot 3\boldsymbol{p} - 2\boldsymbol{q} \cdot 5\boldsymbol{q} \\ = & \; 3|\boldsymbol{p}|^2 + 5\boldsymbol{q} \cdot \boldsymbol{p} - 6\boldsymbol{q} \cdot \boldsymbol{p} - 10|\boldsymbol{q}|^2 \\ = & \end{aligned}$$

Simplify $(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q})$ when $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular



$$
\begin{aligned}
&(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q}) \\
&= 3\boldsymbol{p} \cdot \boldsymbol{p} + 5\boldsymbol{q} \cdot \boldsymbol{p} - 2\boldsymbol{q} \cdot 3\boldsymbol{p} - 2\boldsymbol{q} \cdot 5\boldsymbol{q} \\
&= 3|\boldsymbol{p}|^2 + 5\boldsymbol{q} \cdot \boldsymbol{p} - 6\boldsymbol{q} \cdot \boldsymbol{p} - 10|\boldsymbol{q}|^2 \\
&= 3|\boldsymbol{p}|^2 - \boldsymbol{q} \cdot \boldsymbol{p} - 10|\boldsymbol{q}|^2 \\
&=
\end{aligned}
$$

Simplify $(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q})$ when $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular



$$
\begin{aligned}
&(\boldsymbol{p} - 2\boldsymbol{q}) \cdot (3\boldsymbol{p} + 5\boldsymbol{q}) \\
&= 3\boldsymbol{p} \cdot \boldsymbol{p} + 5\boldsymbol{q} \cdot \boldsymbol{p} - 2\boldsymbol{q} \cdot 3\boldsymbol{p} - 2\boldsymbol{q} \cdot 5\boldsymbol{q} \\
&= 3|\boldsymbol{p}|^2 + 5\boldsymbol{q} \cdot \boldsymbol{p} - 6\boldsymbol{q} \cdot \boldsymbol{p} - 10|\boldsymbol{q}|^2 \\
&= 3|\boldsymbol{p}|^2 - \boldsymbol{q} \cdot \boldsymbol{p} - 10|\boldsymbol{q}|^2 \\
&= 3|\boldsymbol{p}|^2 - 10|\boldsymbol{q}|^2 \\
&(\because \boldsymbol{p} \cdot \boldsymbol{q} = 0)
\end{aligned}
$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$$

$=$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -\begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -\begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -9+3 \\ -1+1 \\ +2+3 \end{pmatrix} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OQ} = -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -\begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} -9+3 \\ -1+1 \\ +2+3 \end{pmatrix} = \begin{pmatrix} -6 \\ 0 \\ 5 \end{pmatrix}$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PR} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\overrightarrow{OP} + \overrightarrow{OR}$$

$$=$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\overrightarrow{OP} + \overrightarrow{OR}$$

$$= -\begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\overrightarrow{OP} + \overrightarrow{OR}$$

$$= -\begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -9+1 \\ -1+0 \\ +2-1 \end{pmatrix} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\overrightarrow{OP} + \overrightarrow{OR}$$

$$= -\begin{pmatrix} 9 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -9+1 \\ -1+0 \\ +2-1 \end{pmatrix} = \begin{pmatrix} -8 \\ -1 \\ 1 \end{pmatrix}$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \left(\overrightarrow{PQ} \;\; \overrightarrow{PR} \right) = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$\overrightarrow{PQ} \times \overrightarrow{PR} =$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \begin{pmatrix} \overrightarrow{PQ} & \overrightarrow{PR} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} +$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

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Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 5 & 1 \\ -6 & -8 \end{vmatrix} \boldsymbol{j} +$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

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$$=$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \begin{pmatrix} \overrightarrow{PQ} & \overrightarrow{PR} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 5 & 1 \\ -6 & -8 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} -6 & -8 \\ 0 & -1 \end{vmatrix} \boldsymbol{k}$$

$$= \{0 \cdot (1) - (-1) \cdot 5\} \boldsymbol{i} +$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \begin{pmatrix} \overrightarrow{PQ} & \overrightarrow{PR} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 5 & 1 \\ -6 & -8 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} -6 & -8 \\ 0 & -1 \end{vmatrix} \boldsymbol{k}$$

$$= \{0 \cdot (1) - (-1) \cdot 5\}\boldsymbol{i} + \{5 \cdot (-8) - (1) \cdot (-6)\}\boldsymbol{j}$$

$$+$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

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$$= \{0 \cdot (1) - (-1) \cdot 5\}\boldsymbol{i} + \{5 \cdot (-8) - (1) \cdot (-6)\}\boldsymbol{j}$$

$$+\{-6 \cdot (-1) - (-8) \cdot (0)\}\boldsymbol{k} =$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \begin{pmatrix} \overrightarrow{PQ} & \overrightarrow{PR} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 5 & 1 \\ -6 & -8 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} -6 & -8 \\ 0 & -1 \end{vmatrix} \boldsymbol{k}$$

$$= \{0 \cdot (1) - (-1) \cdot 5\}\boldsymbol{i} + \{5 \cdot (-8) - (1) \cdot (-6)\}\boldsymbol{j}$$

$$+\{-6 \cdot (-1) - (-8) \cdot (0)\}\boldsymbol{k} = \{0 + 5\}\boldsymbol{i} +$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \begin{pmatrix} \overrightarrow{PQ} & \overrightarrow{PR} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 5 & 1 \\ -6 & -8 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} -6 & -8 \\ 0 & -1 \end{vmatrix} \boldsymbol{k}$$

$$= \{0 \cdot (1) - (-1) \cdot 5\}\boldsymbol{i} + \{5 \cdot (-8) - (1) \cdot (-6)\}\boldsymbol{j}$$

$$+\{-6 \cdot (-1) - (-8) \cdot (0)\}\boldsymbol{k} = \{0 + 5\}\boldsymbol{i} + \{-40 + 6\}\boldsymbol{j}$$

$$+$$

Points P, Q, and R have coordinates $(9, 1, -2)$, $(3, 1, 3)$, and $(1, 0, -1)$ respectively. Find $\overrightarrow{PQ} \times \overrightarrow{PR}$.

$$\therefore \begin{pmatrix} \overrightarrow{PQ} & \overrightarrow{PR} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ 0 & -1 \\ 5 & 1 \end{pmatrix}$$

Thus

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} 0 & -1 \\ 5 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 5 & 1 \\ -6 & -8 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} -6 & -8 \\ 0 & -1 \end{vmatrix} \boldsymbol{k}$$

$$= \{0 \cdot (1) - (-1) \cdot 5\}\boldsymbol{i} + \{5 \cdot (-8) - (1) \cdot (-6)\}\boldsymbol{j}$$

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Find a vector $\boldsymbol{c}$ which is perpendicular to both of the vectors $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{b} = -\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$

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$$\left(\begin{array}{cc} \boldsymbol{a} & \boldsymbol{b} \end{array} \right) \Longrightarrow \left(\begin{array}{cc} 1 & -1 \\ 1 & -1 \\ 0 & 1 \\ 1 & -1 \end{array} \right)$$

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$$\left(\boldsymbol{a} \quad \boldsymbol{b} \right) \Longrightarrow \begin{pmatrix} 1 & -1 \\ 1 & -1 \\ 0 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} \boldsymbol{i} + \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} \boldsymbol{k} = \boldsymbol{i} - \boldsymbol{j}$$

$\therefore$

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$$\therefore \boldsymbol{c} = \alpha \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$ **where α is any real number**