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ON IMPLEMENTING A NUMERIC HUYGEN'S SOURCE SCHEME IN A FINITE DIFFERENCE PROGRAM TO ILLUMINATE SCATTERING BODIES

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ABSTRACT

A numerical procedure is described that will simplify the analysis of the EMP response of structures with dielectric or poorly conducting segments.

I. Introduction

Many of the problems of interest in EMP studies involve configurations of metals and dielectrics or poorly conducting materials. Some examples are:

1. Composite airplanes;
2. Metal airplanes on a dielectric platform;
3. Missiles with dielectric motor bottles;
4. Shelters half buried in poorly conducting earth;
5. Missiles with ionized exhaust plumes.

These problems are difficult to treat with the usual finite difference computer programs.

In the study of the response of metal bodies in a plane wave field, we compute only the scattered field outside the body. The boundary condition that $E^{tan} = 0$ provides the forcing function for the problem: $E^{scat} = - E^{inc}$ on the surface of the body.

The exterior problem space boundary conditions that are used to terminate typical finite difference grids all make use of the fact that the scattered electromagnetic fields at the edge of the grid are all traveling away from the body.

Whenever the problem space contains some dielectric material with $\varepsilon \neq \varepsilon_o$ (or a poorly conducting material), the situation is complicated in that we do not a-priori know the scattered fields at the outer surface of the dielectric scatterer.

The introduction of equivalent sources into the problem space appears to be an effective way to deal with this problem. It is this approach that will be described here.

II. The Equivalence Theorem

The equivalence theorem is one of the fundamental theorems used in theoretical electrodynamics. It is based on Huygen's principle which stated simply that "each point on a wave front acts like a new source of waves." The extension to electromagnetics which provides the precise definition of these sources is the equivalence theorem, although the equivalent source current densities are sometimes called Huygen's sources.[1] To develop this idea, let us consider a source radiating a field from Region 2 into Region 1 which are separated by an artificial surface S (Figure 1). Let us now imagine that electric and magnetic surface currents flow on the artificial surface. The tangential electric and magnetic fields are then discontinuous across the surface:[2]

$$\vec{n} \times (\vec{H}^1 - \vec{H}^2) = \vec{J}_s \qquad (1)$$

$$(\vec{E}^1 - \vec{E}^2) \times \vec{n} = \vec{M}_s \qquad (2)$$

where $\vec{n}$ is a unit vector normal to the surfaces and directed from Region 2 into Region 1. We may now define the Huygen's source currents by insisting that both $\vec{E}^2$ and $\vec{H}^2$ be zero, and that $\vec{E}^1$ and $\vec{H}^1$ be exactly those fields produced by the original sources at the boundary:

$$\vec{M}_s^h = \vec{E}^{inc} \times \vec{n} \qquad (3)$$

$$\vec{J}_s^h = \vec{n} \times \vec{H}^{inc} \qquad (4)$$

From the uniqueness theorem, we know that if the fields produced by the Huygen's sources are correct everywhere on the boundary then they are correct everywhere in the confined region (Figure 1).

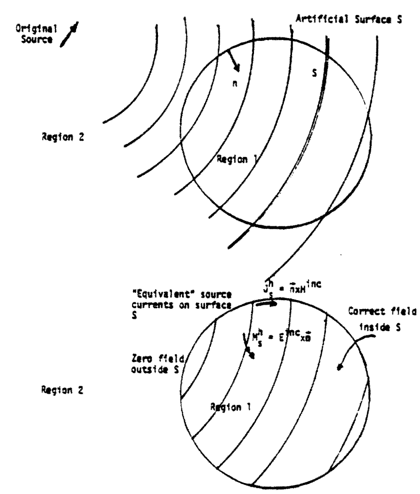

Figure 1. Equivalent Source Currents Produce Correct Interior Fields.

The application of the equivalence principle to scattering problems is straightforward. Conceptually, we remove the scatterer from the problem space and compute the fields that would exist inside the Huygen's surface without the scatterer present. From these fields the Huygen's source current densities are determined. These source currents produce the correct incident field inside the Huygen's surface and zero field outside the surface. When the scatterer is

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reintroduced, the fields inside the Huygen's surface are the total fields (incident and scattered) and the fields outside the Huygen's surface are just the scattered fields. The reader will note that there is no reflection of the scattered field by the artificially introduced Huygen's surface since this surface is just a location for electric and magnetic current sources which do not depend upon the scattered field.

While we may initially envision that the region inside the Huygen's surface is homogeneous when the scatterer is not present, there is no requirement that it be. For example, the treatment of airplanes parked on a lossy ground plane requires that the earth be present when the incident field is initially determined. The resulting Huygen's source currents are then distributed both above and below the surface of the earth. The correct field inside the Huygen's surface will be obtained only when the earth is present throughout the entire half plane represented by the ground, that is both inside and outside the Huygen's surface.

III. Finite Difference Implementation in 3-Dimensions

Implementing the equivalence principle into a finite difference computer program is complicated by the fact that H and E nodes are at different points introducing some uncertainty as to what should be the time displacement between electric and magnetic current sources. Here we develop the solution for a 3-D rectangular coordinate system (Figure 2). Conceptually,

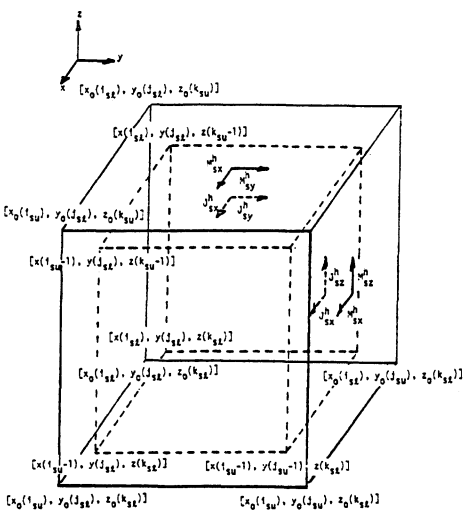

Figure 2. Definition of 3-D Boxes Where Huygen's Equivalent Source Currents are Defined.

we can visualize two rectangular boxes: the outer box is defined by locations where the Huygen's electric current sources are to be defined. When the problem space contains only these sources, we expect that both electric and magnetic fields inside and on the surface of the outer box are zero. The finite difference form of the Huygen's sources are developed directly from the finite difference form of Maxwell's curl equations applied at the approximate surface grid points. For the left surface of the outer box, we obtain:[3]

$$-\mu \frac{\partial H_x}{\partial t} = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + M^h_x$$

$$-\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + M^h_z$$

or in finite difference form

$$H^{n+1}_x(i,j_{s\ell},k) = H^n_x(i,j_{s\ell},k) - \frac{\Delta t}{\mu} \frac{E^n_z(i,j_{s\ell},k) - E^n_z(i,j_{s\ell}-1,k)}{y(j_{s\ell}) - y(j_{s\ell}-1)} + \frac{\Delta t}{\mu} \frac{E^n_y(i,j_{s\ell},k) - E^n_y(i,j_{s\ell},k-1)}{z(k) - z(k-1)} - \frac{\Delta t}{\mu} M^h_x(i,j_{s\ell},k) \quad (6)$$

for $i\varepsilon[i_{s\ell},i_{su}-1]$, and $k\varepsilon[k_{s\ell},k_{su}]$

and

$$H^{n+1}_z(i,j_{s\ell},k) = H^n_z(i,j_{s\ell},k) - \frac{\Delta t}{\mu} \frac{E^n_y(i,j_{s\ell},k) - E^n_y(i-1,j_{s\ell},k)}{x(i) - x(i-1)} + \frac{\Delta t}{\mu} \frac{E^n_x(i,j_{s\ell},k) - E^n_x(i,j_{s\ell}-1,k)}{y(j_{s\ell}) - y(j_{s\ell}-1)} - \frac{\Delta t}{\mu} M^h_z(i,j_{s\ell},k) \quad (7)$$

for $i\varepsilon[i_{s\ell},i_{su}]$, and $k\varepsilon[k_{s\ell},k_{su}-1]$.

The fields created by Huygen's sources are zero outside and on the outer surface, therefore, in Eq. (6),

$$H_x(i,j_{s\ell},k) = E_y(i,j_{s\ell},k) = E_z(i,j_{s\ell}-1,k) = 0 \quad (8)$$

and in Eq. (7)

$$H_z(i,j_{s\ell},k) = E_y(i,j_{s\ell},k) = E_x(i,j_{s\ell}-1,k) = 0$$

while the field inside and on the inner surface is the desired incident field evaluated at the point in question:

$$E^n_z(i,j_{s\ell},k) = E^{inc}_z(t_E(n),x(i),y(j_{s\ell}),z_o(k)) \quad (10)$$

and

$$E^n_x(i,j_{s\ell},k) = E^{inc}_x(t_E(n),x_o(i),y(j_{s\ell}),z(k)) \quad . \quad (11)$$

Therefore, the source terms M^h_x and M^h_z are easily identified.

$$M^h_x(i,j_s\ ,k) = -\left(\frac{E^{inc}_z(t_E(n),x(i),y(j_{s\ell}),z_o(k))}{y(j_{s\ell}) - y(j_{s\ell}-1)}\right) \quad (12)$$

for $i\varepsilon[i_{s\ell},i_{su}-1]$, and $k\varepsilon[k_{s\ell},k_{su}]$

and

$$M_z^h(i,j_{s\ell},k) = + \left(\frac{E_x^{inc}(t_E(n),x_o(i),y(j_{s\ell}),z(k))}{y(j_{s\ell}) - y(j_{s\ell}-1)} \right) \quad (13)$$

for $i\varepsilon[i_{s\ell},i_{su}]$, and $k\varepsilon[k_{s\ell},k_{su}-1]$

Formulas for the electric source currents are obtained from the finite difference form of the curl equation used to determine the electric field. The tangential electric fields on the left surface of the inner box are computed from the curl of $\vec{H}$:

$$\sigma E_x + \varepsilon \frac{\partial E_x}{\partial t} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) - j_x^h \quad (14)$$

$$\sigma E_z + \varepsilon \frac{\partial E_z}{\partial t} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) - j_z^h$$

or in finite difference notation:

$$B \cdot E_x^{n+1}(i,j_{s\ell},k) = A \cdot E_x^n(i,j_{s\ell},k) - J_x^h(i,j_{s\ell},k) + \left(\frac{H_z^{n+1}(i,j_{s\ell}+1,k) - H_z^{n+1}(i,j_{s\ell},k)}{y_o(j_{s\ell}+1) - y_o(j_{s\ell})} \right) - \left(\frac{H_y^{n+1}(i,j_{s\ell},k+1) - H_y^{n+1}(i,j_{s\ell},k)}{z_o(k+1) - z_o(k)} \right) \quad (15)$$

for $i\varepsilon[i_{s\ell}+1,i_{su}-1]$, and $k\varepsilon[k_{s\ell},k_{su}-1]$

where, $B = \frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}$ $\qquad A = \frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}$

and

$$B \cdot E_z^{n+1}(i,j_{s\ell},k) = A \cdot E_z^n(i,j_{s\ell},k) - J_z^h(i,j_{s\ell},k) + \left(\frac{H_y^{n+1}(i+1,j_{s\ell},k) - H_y^{n+1}(i,j_{s\ell},k)}{x_o(i+1) - x_o(i)} \right) - \left(\frac{H_x^{n+1}(i,j_{s\ell}+1,k) - H_x^{n+1}(i,j_{s\ell},k)}{y_o(j_{s\ell}+1) - y_o(j_{s\ell})} \right) \quad (16)$$

for $i\varepsilon[i_{s\ell},i_{su}-1]$, and $k\varepsilon[k_{s\ell}+1,k_{su}-1]$.

We solve for the unknown electric source currents by noting that in these equations, all the fields are supposed to be equal to the incident field with the exception of those magnetic field components that are located on the outside surface:

$$H_z^{n+1}(i,j_{s\ell},k) = H_x^{n+1}(i,j_{s\ell},k) = 0 \quad .$$

On this left surface, the electric source currents are chosen to exactly replace the missing magnetic fields in Eq. (15) and (16) so that the correct values of the incident E_x and E_z components will be computed.

$$J_x^h(i,j_{s\ell},k) = \left(\frac{H_z^{inc}(t_H(n+1),x_o(i),y_o(j_{s\ell}),z(k))}{y_o(j_{s\ell}+1) - y_o(j_{s\ell})} \right) \quad (17)$$

for $i = [i_{s\ell}+1,i_{su}-1]$, and $k = [k_{s\ell},k_{su}-1]$

$$J_z^h(i,j_{s\ell},k) = - \left(\frac{H_x^{inc}(t_H(n+1),x(i),y_o(j_{s\ell}),z_o(k))}{y_o(j_{s\ell}+1) - y_o(j_{s\ell})} \right) \quad (18)$$

for $i = [i_{s\ell},i_{su}-1]$, and $k = [k_{s\ell}+1,k_{su}-1]$.

The development given above is only for one of the six faces of the Huygen's surface but the form of the solution is apparent. That is,

$$\vec{M}^h = \frac{\vec{E}^{inc} \times \vec{n}}{\Delta s} \quad (19)$$

and

$$\vec{J}^h = \frac{\vec{n} \times \vec{H}^{inc}}{\Delta s_o} \quad (20)$$

Here $\vec{n}$ is the inward normal:

Δs_o is the cell size in the $\vec{n}$ direction at the boundary, evaluated at tangential magnetic field points;

Δs is the cell size in the $\vec{n}$ direction at the boundary, evaluated at tangential electric field points;

$\vec{E}^{inc} \times \vec{n}$ is computed on the inner surface; and

$\vec{n} \times \vec{H}^{inc}$ is computed on the outer surface.

From Eq. (19) and (20), the formulas for Huygen's sources on any surface can be determined. The full set of equations for the six Huygen's surfaces implicit in Eq. (19) and (20) have been used as the basis of a three dimensional Huygen's surface algorithm. Example routines in either two or three dimensions can be obtained from the authors upon request. Figure 3 shows the result obtained when a plane wave is propagated across a grid using this scheme. You will note that the fields outside the Huygen's surface are very small. In fact, it appears that the accuracy of the Huygen's source implementation is just the accuracy of the finite difference procedure itself.

When a scatterer is present inside the problem space, the field inside the Huygen's surface is the total field while the field outside is the scatterer field. Since the scatterer field is an outward going wave, the problem space boundary conditions are more accurate.

IV. Finite Difference Implementation in 2-Dimensions

For some problems 2-dimensional finite difference programs are useful. For example, to examine how the plume conductivity profile affects the total current flowing on a missile at low frequencies, a 2-D cylindrical coordinate computer program (that computes E_z, E_r, and H_ϕ) would be suitable.

In this case, the Huygen's surface as implemented in the finite difference program is a pair of concentric cannisters with magnetic currents defined on the outside surface (where H_ϕ is zero) and electric

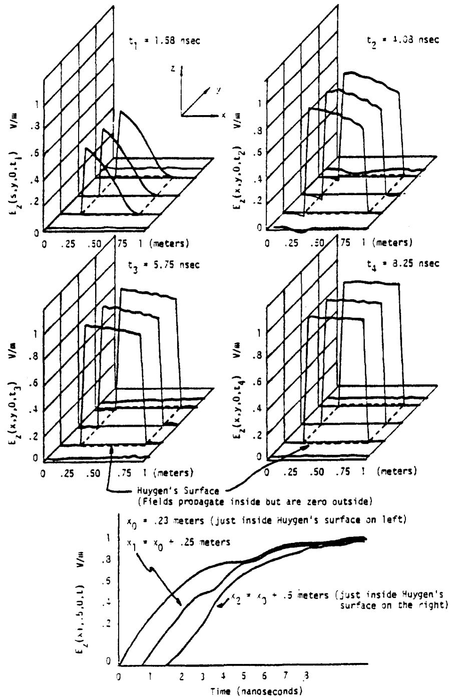

Figure 3. Empty Grid Example for 3-D Rectangular Grid with $E^{inc}=E_0(t-x/c)\vec{a}_z, E_0(\tau) = (1.-\exp.(-\tau/2x10^{-9}))\ \Delta x = \Delta y = 0.05$ meters.

currents defined on the inside surface. Along the top and bottom of the outside cannister the magnetic current density is from (19):

$$\vec{M}^h(r,z) = \frac{1}{2\pi\Delta s_E}\int_0^{2\pi} E^{inc}(r,\phi,z) \times \vec{n}\ d\phi \qquad (21)$$

and the electric current density on the inner cannister is:

$$\vec{J}^h(r,z) = \frac{1}{2\pi\Delta s_H}\int_0^{2\pi} \vec{n} \times H^{inc}(r,\phi,z)\ d\phi \qquad (22)$$

where $\vec{n}$ is always an inward normal:

Δs_E and Δs_H are the grid spacing in the $\vec{n}$ direction, evaluated at E nodes and H nodes, respectively.

A listing of a program to accomplish this integration is given in the Appendix.

In Figure 4 is shown an empty grid example for the 2-D case when the incident field is a plane wave with $E^{inc} = E_0(t-x/c)\vec{a}_z$. In this case the risetime of the electric field at the outer edge of the problem space is slower than it is in the center of the grid. This is caused by the averaging that is done around the circumference. However, as the wave propagates in toward the center, the risetime decreases so that at the center of the grid it is very similar to the input field.

In practice it will be desirable to place the Huygen's surface as close to the scatterer as possible to minimize the computational errors.

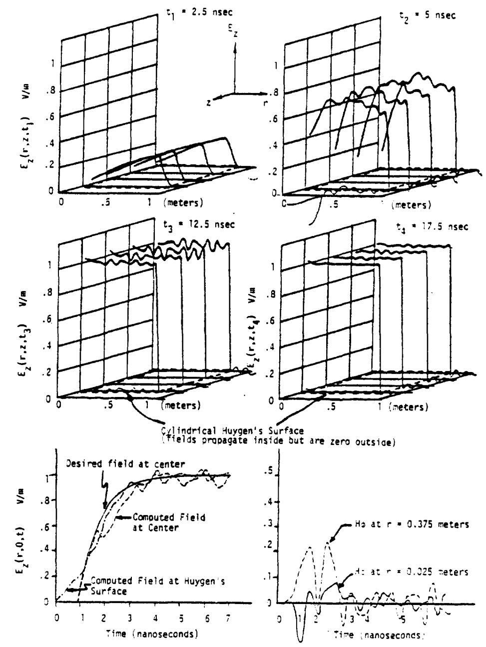

Figure 4. Empty Grid Example of 2-D Cylindrical Grid with $E^{inc}=E_0(t-x/c)\vec{a}_z$, $E_0(\tau) = (1.-\exp(-\tau/2x10^{-9}))\ \Delta z = \Delta z = 0.05$ meters.

V. Source Region EMP

In the source region of a nuclear burst, there may be both locally generated EM fields as well as propagated fields.[4] To address problems in the source region Compton currents should be included inside the Huygen's surface (Figure 1), however, no change in the definition of J_s^h and M_s^h are needed. It should be recognized however that the Huygen's sources so defined both "add-in" contribution from exterior sources to the enclosed volume, and "cancel-out" exterior fields emanating from the interior sources.

VI. Conclusions

The equivalence principle is one of the basic principles of electrodynamics, however, it has not been used before in finite difference computer programs. We have found that it is easily implemented and is surprisingly accurate even in the degenerate 2-dimensional case.

The authors have utilized this approach to study the EMP coupling to the MX missile, scattering from the wooden TRESTLE in the ATLAS I simulator, and to evaluate the effects of the wooden platform on aircraft response at HPD. We have found it is also useful for more accurately predicting the surface currents and charge densities on the shadowed side of metal airplanes. This results because the total field is computed directly; addition of the numerically calculated scattered field to an analytical incident field is not required.

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