Automating Prover Development for Non-Classical Logics

Renate A. Schmidt

School of Computer Science
The University of Manchester

6 September 2013

Outline

1. Automated prover generation
2. The MetTeL tool
3. Termination via blocking
4. Rule refinement

Introduction

> The toolset of every logician, and everyone using logic, should include at least one prover

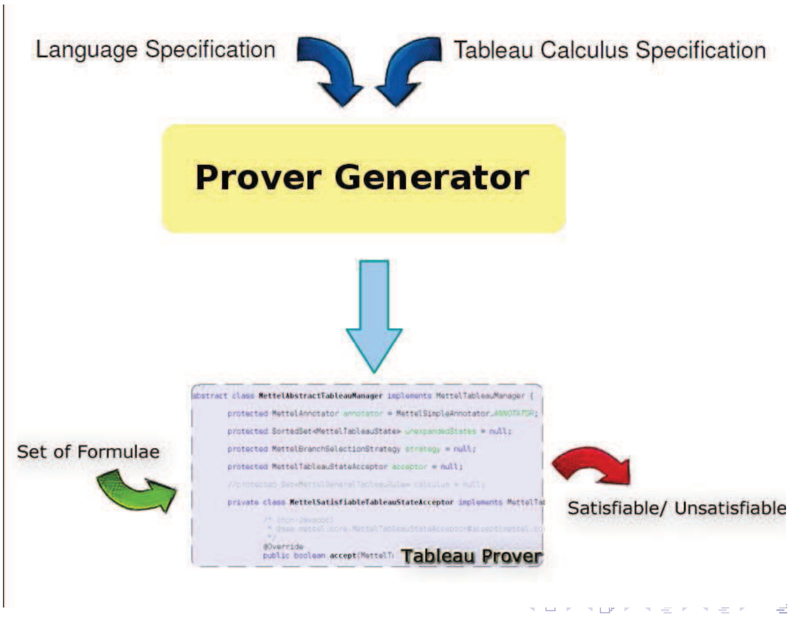
- Too few people use provers and too few provers are available
- Implementing a prover is not straightforward and time-consuming

Solution

Use **prover engineering platforms** or **prover generators**


LoTREC   The Tableau Work Bench   METTEL

Automated prover generation



The MetTeL project

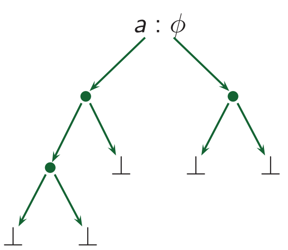
EPSRC project
Started 2011
Dmitry Tishkovsky, Mohammad Khodadadi

Design objectives

- Easy to use
- General and flexible
- Reliable
- Easily extended and integrated

Semantic labelled tableau

- Aim: construct a model or refute given formulae
- Goal-directed
- Rules break down formulae
- Rules for each logical operator
- Branching rules $\rightsquigarrow$ derivations are trees



$$\frac{s : \psi_1 \wedge \psi_2}{s : \psi_1,\ s : \psi_2} \qquad \frac{s : \psi_1 \vee \psi_2}{s : \psi_1 \mid s : \psi_2}$$

$$\frac{s : \Box\psi,\ (s,t) : R}{t : \psi}$$

$$\frac{s : \neg\Box\psi}{(s,u) : R,\ u : \neg\psi}\ (u \text{ new})$$

etc

Using MetTeL to generate prover for S4

S4 = modal logic K in which the accessibility relation R is a pre-order

```
// Input file for Mettel
// to generate prover for modal logic S4
specification S4;

options{
    name.separator=
    // tableau.rule.delimiter=;
    // tableau.rule.branch.delimiter=||
    // tableau.rule.premise.delimiter=/
    // list.left.delimiter=
    // list.right.delimiter=
    // branch.bound=
}
```


Input file for MetTeL (cont'd): The tableau language

```
syntax S4 { // tableau language for logic S4
    sort formula, world;      // declare two sorts

    // tableau formulae are labelled formulae
    formula at = '@' world formula;

    // connectives of the logic
    formula true = 'true';
    formula negation = '~' formula;
    formula box = '[]' formula;
    formula disjunction = formula '|' formula;
```

- Any connectives of fixed finite arity can be defined

Input file for MetTeL (cont'd): The tableau language

```
    // explicit accessibility relation
    formula relation = 'R' '(' world ',' world ')';

    // witnesses for diamond formulae
    world   succ = 'f' '(' world ',' formula  ')';

    // for blocking
    formula equality = '[' world '=' world ']';
}
```

- **equality** = MetTeL keyword, used to define equality symbol in the tableau language

Input file to MetTeL (cont'd): The tableau calculus

```
tableau S4 {  // tableau calculus
    @s P  @s (~P)            // Closure rule
      /                            priority 0 $;
    @s (~true)               // Decomposition rules
      /                            priority 1 $;
    @s (~(~P))
      /  @s P                      priority 1 $;
    @s (P|Q)
      /  @s P  $|  @s Q            priority 3 $;
    @s (~(P|Q))
      /  @s (~P)  @s (~Q)          priority 1 $;
    @s ([]P)  R(s,t)
      /  @t P                      priority 2 $;
    @s (~([]P))
      /  R(s,f(s,P))  @f(s,P) (~P)  priority 7 $;
           // f(s,P) is newly created R-successor
```

Input to MetTeL (cont'd): The tableau calculus

```
    @s P                          // Reflexivity
       /  R(s,s)                           priority 1 $;
    R(s,t)  R(t,u)                // Transitivity
       /  R(s,u)                           priority 2 $;

    @s P    @t Q                  // Blocking rule
       /  [s=t]  $|  ~([s=t])              priority 6 $;
//     R(s,t)                       // Ancestor blocking
//        /  [s=t]  $|  ~([s=t])           priority 6 $;

}
```


Running MetTeL

```
> java -jar mettel2.jar -i ~/work/mettel/S4/spec
```

creates the prover.

Running the generated prover:

```
> java -jar S4.jar
```

with

```
@s p
@s (~p)
```

produces

```
Unsatisfiable.
Contradiction: [( @ s p ), ( @ s ( ~ p ) )]
```

Web interface www.mettel-prover.org

- Modal Logics
- Description logics, incl. $\mathcal{ALBO}^{id}$
- Intuitionistic propositional logic
- Three valued Lukasiewicz logic
- Simple properties of lists
- Hybrid modal logics & counting operators
- Multi-agent interrogative-epistemic logics with privacy & sequential queries
- Temporal logic
- Testing rule admissibility
- ...

Additional features and functionalities

- Rule applications are controlled via rule priorities
- Equality reasoning via ordered term rewriting
- DFLR and BF search strategies
- Generic blocking mechanism
- Dynamic backtracking
- Random formula generation
- Conflict directed backjumping
- Support for benchmarking

Generic blocking mechanism

General idea of blocking

Use the tableau procedure to find finite models through identifying terms or reusing terms

Unrestricted blocking

$$\text{(ub)} \qquad \frac{}{s \approx t \;\mid\; s \not\approx t}$$

$s \approx t$ is a trigger for ordered rewriting $s \longrightarrow t$, if $s \succ t$

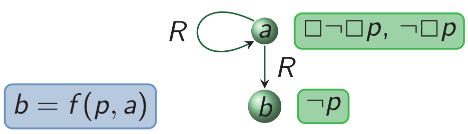
Termination condition
Apply (ub) rule eagerly for all distinct pairs of terms

Example using unrestricted blocking

$$\text{(ub)} \qquad \frac{}{s \approx t \;\mid\; s \not\approx t}$$

$a : \Box\neg\Box p$
$R(a, a)$
$a : \neg\Box p$
$R(a, b)$
$b : \neg p$
$a \approx b$

Example: $\Box\neg\Box p$ is $S4$-satisfiable



Assume $a \approx b$
Rewrite $b \longrightarrow a$

Example using unrestricted blocking

(ub) $\dfrac{}{s \approx t \mid s \not\approx t}$

Example: $\Box\neg\Box p$ is *S4*-satisfiable

$\Box\neg\Box p,\ \neg\Box p,\ \neg p$

$b = f(p, a)$

$a \approx b$

$a : \Box\neg\Box p$
$R(a, a)$
$a : \neg\Box p$
$R(a, b)$
$b : \neg p$
$a \approx b$

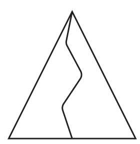
$a : \Box\neg\Box p$
$R(a, a)$
$a : \neg\Box p$
$\cancel{R(a, a)}$
$a : \neg p$
$a \approx b$

Soundness, completeness and termination

Theorem

- $Tab + (ub)$ is a sound and complete, if *Tab* is sound and complete.
- $Tab + (ub)$ is terminating, if the logic has the finite model property.

Tab is *terminating* if for any *finite* set N, every open tableau $Tab(N)$ has a finite open branch.



Semantic labelled tableaux can decide

- Numerous modal, description and hybrid logics, incl. $\mathcal{ALBO}^{id}$
- Two-variable fragment of first-order logic

Restricting the application of blocking

Adding premises/side-conditions restricts the application of the (ub) rule

Predecessor blocking

$$\text{(ub-pred)} \qquad \frac{R(s,t)}{s \approx t \mid s \not\approx t}$$

Gives ancestor blocking for S4

Theorem

- *Tab* extended with any such restricted blocking rule is sound and complete, if *Tab* is sound and complete.

Exclude a set from blocking

Let S be a finite set of terms

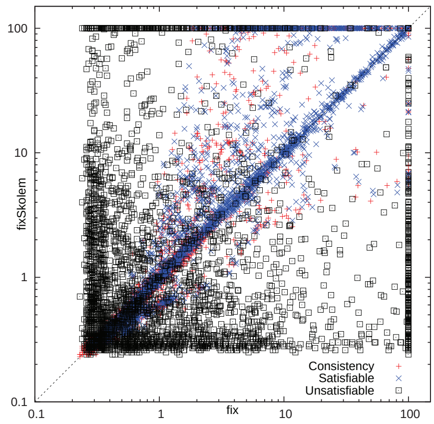
$$\text{(ub-noS)} \qquad \frac{}{s \approx t \mid s \not\approx t} \quad t \notin S$$

Theorem

- *Tab* + (ub-noS) is sound, complete and terminating,
if *Tab* is sound and complete and *Tab* +(ub) is terminating.

Unrestricted blocking vs. ub-noS

- Experimental results for description logic SHOI
- Manchester corpus: ∼4500 OWL ontologies
- S = ABox elements



Rule refinement

Moving formulae from conclusion positions to premise positions

- Reduces branching
- Search space is smaller

$$\frac{s : \Box\phi}{\neg R(s,t) \mid t : \phi} \quad \text{refines to} \quad \frac{s : \Box\phi,\ R(s,t)}{t : \phi}$$

$$\frac{}{\neg R(s,t) \mid \neg R(t,u) \mid R(s,u)} \quad \text{refines to} \quad \frac{R(s,t),\ R(t,u)}{R(s,u)}$$

$$\frac{s : \phi \vee \psi}{s : \phi \mid s : \psi} \quad \text{refines to} \quad \frac{s : \phi \vee \psi,\ s : \neg\phi}{s : \psi}$$

Atomic rule refinement

Theorem

- Replacing a rule by its atomic rule refinement preserves soundness and completeness. Holds for any semantic tableau calculus.

Moving negated atoms to premise positions

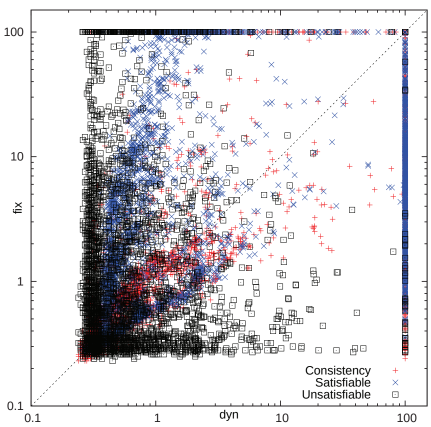
$$\frac{s : \neg p \vee \psi}{s : \neg p \;\mid\; s : \psi} \qquad \text{replaced by} \qquad \frac{s : \neg p \vee \psi, \;\; s : p}{s : \psi}$$

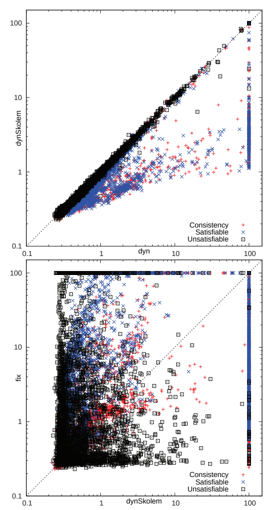
Separate rules for all TBox statements $\quad A \sqsubseteq \forall R.B$

$$\frac{}{s : \neg A \sqcup \forall R.B} \qquad \text{replaced by} \qquad \frac{s : A, \;\; R(s, t)}{t : B}$$

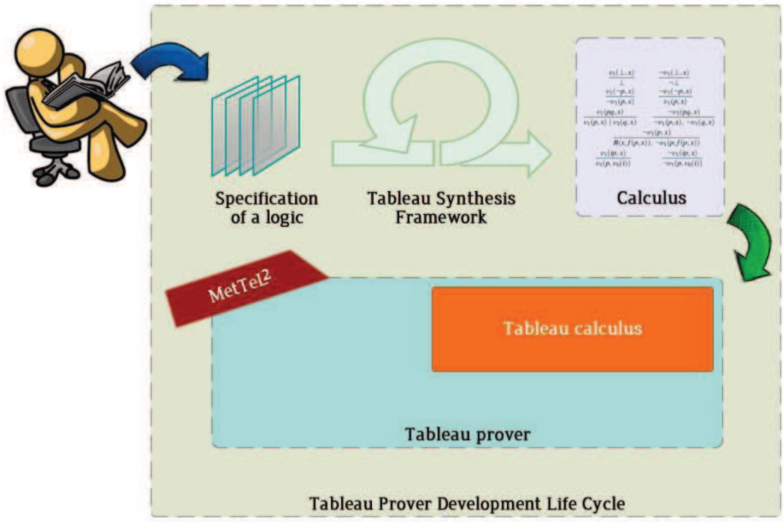
More experimental results for SHOI

Effect of atomic rule refinement for TBox statements





The vision



Ongoing and future work

- Implement tableau calculus synthesis
- Study restricted forms of blocking; decidability; automatic use of appropriate blocking
- Stronger conditions/principles for automated rule refinement
- Incorporate techniques for automatically computing correspondence properties

Thank you!

- **Dmitry Tishkovsky**
- **Mohammad Khodadadi**
- Fabrizio Principato
- Jacobus Meulen
- Tomas Alijevas

- Michal Zawidzki
- Stefan Minica
- Clare Dixon, Boris Konev
- John Stell, David Rydeheard