Normal Forms and Proofs in Combined Modal and Temporal Logics

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Abstract. In this paper we present a framework for the combination of modal and temporal logic. This framework allows us to combine different normal forms, in particular, a separated normal form for temporal logic and a first-order clausal form for modal logics. The calculus of the framework consists of temporal resolution rules and standard first-order resolution rules.
We show that the calculus provides a sound, complete, and terminating inference systems for arbitrary combinations of subsystems of multimodal S5 with linear, temporal logic.

1 Introduction

For a number of years, temporal and modal logics have been applied outside pure logic in areas such as formal methods, theoretical computer science and artificial intelligence. A variety of sophisticated methods for reasoning with these logics have been developed, and in many cases applied to real-world problems.

With the advent of more complex applications, a *combination* of modal and temporal logics is increasingly required, particularly in areas such as security in distributed systems (Halpern 1987), specifying multi-agent system (Jennings 1999; Wooldridge and Jennings 1995), temporal databases (Finger 1994), and accident analysis (Johnson 1994). Further motivation for the importance of combinations of modal logics can be found in (Blackburn and de Rijke 1997).

In all these cases, combinations of multi-modal and temporal logics are used to capture the detailed behaviour of the application domain. While many of the basic properties of such combinations are well understood (Baader and Ohlbach 1995; Fagin et al. 1996; Gabbay 1996; Wolter 1998), very little work has been carried out on proof methods for such logics. Wooldridge, Dixon, and Fisher (1998) present a tableaux-based calculus for the combination of discrete linear temporal logic with the modal logics KD45 (characterising belief) and S5 (characterising knowledge). Dixon, Fisher, and Wooldridge (1998) present a resolution-based calculus for the combination of discrete linear temporal logic with the modal logic S5. A combination of calendar logic for specifying everyday temporal notions with a variety of other modal logics has been considered in (Ohlbach 1998; Ohlbach and Gabbay 1998).

Our aim in this paper is to present an approach that is general enough to capture a wide range of combinations of temporal and modal logics, but still provides viable means for effective theorem proving. The following aspects of our approach are novel:

- The approach covers the combination of discrete, linear, temporal logic with extensions of multi-modal K_m by any combination of the axiom schemata 4, 5, B, D, and T. This extends the results presented in (Dixon et al. 1998; Wooldridge et al. 1998).
- Instead of combining two calculi operating according to the same underlying principles, like for example two tableaux-based calculi, we combine two different approaches to theorem-proving in modal and temporal logics, namely the translation approach for modal logics (using first-order resolution) and the SNF approach for temporal logics (using modal resolution).
- The particular translation we use has only recently been proposed by de Nivelle (1999) and can be seen as a special case of the T-encoding introduced by Ohlbach (1998). It allows for conceptually simple decision procedures for extensions of K4 by ordered resolution without any reliance on loop checking or similar techniques.

2 Combinations of temporal and modal logics

In this section, we give the syntax and semantics of a class of combinations of a discrete linear temporal logic with normal multi-modal logics.

Syntax

Let P be a set of propositional variables, A a set of agents, and M a set of modalities. Then the tuple $\Sigma = (\mathsf{P}, \mathsf{A}, \mathsf{M})$ is a *signature* of MTL. If $m \in \mathsf{M}$ and $a \in \mathsf{A}$, then the ordered pair (m, a) is a *modal parameter*. The set of *well-formed formulae* of MTL over a signature Σ is inductively defined as follows: (i) true and false are formulae, (ii) every propositional variable is a formula, (iii) if φ and ψ are formulae, then $\neg\varphi$, $\varphi \vee \psi$, $\varphi \wedge \psi$, $\varphi \Rightarrow \psi$, and $\varphi \Leftrightarrow \psi$ are formulae, (iv) if φ and ψ are formulae, then $\bigcirc\varphi$, $\Diamond\varphi$, $\Box\varphi$, $\varphi\mathcal{U}\psi$, and $\varphi\mathcal{W}\psi$ are formulae, (v) if φ is a formula, κ is a modal parameter, then $[\kappa]\,\varphi$ is a formula.

We also use true and false to denote the empty conjunction and disjunction, respectively. A literal is either p or $\neg p$ where p is a propositional variable. A *modal literal* is either $[\kappa]\,L$ or $\neg\,[\kappa]\,L$ where κ is a modal parameter and L is a literal. For any (modal) literal L we denote by $\sim\!L$ the negation normal form of $\neg L$.

With each modal parameter we can associate a set of axiom schemata defining its properties. We assume that the axiom schemata K and Nec hold for every modal operator. In this paper we allow in addition any combination of the axiom schemata 4, 5, B, D, and T.

Semantics

Let $\mathcal{S}$ be a set of states. A *timeline* is an infinite, linear, discrete sequence of states indexed by the natural numbers. A *point* is an ordered pair (t, k) where t is a timeline and k is a natural number, a so-called *temporal index*. $\mathcal{P}$ denotes the set of all points. A *valuation* ι is a mapping from $\mathcal{P}$ to a subset of P. An *interpretation* $\mathcal{M}$ is a tuple $(\mathcal{T}, t_0, \mathcal{R}, \iota)$ where $\mathcal{T}$ is a set of timelines with a distinguished timeline $t_0 \in \mathcal{T}$, $\mathcal{R}$ is a collection of binary relations on $\mathcal{P}$ containing for every modal parameter κ a relation R_κ, and ι is a valuation.

We define a binary relation $\models$ between a formula φ and a pair $\mathcal{M}$ and (t, k) where $\mathcal{M}$ is an interpretation and (t, k) is a point as follows.

$$
\begin{array}{ll}
\mathcal{M},(t,k) \models \mathsf{true} & \\
\mathcal{M},(t,k) \not\models \mathsf{false} & \\
\mathcal{M},(t,k) \models \mathsf{start} & \text{iff } t = t_0 \text{ and } k = 0 \\
\mathcal{M},(t,k) \models p & \text{iff } p \in \iota((t,k)) \\
\mathcal{M},(t,k) \models \neg\varphi & \text{iff } \mathcal{M},(t,k) \not\models \varphi \\
\mathcal{M},(t,k) \models \varphi \wedge \psi & \text{iff } \mathcal{M},(t,k) \models \varphi \text{ and } \mathcal{M},(t,k) \models \psi \\
\mathcal{M},(t,k) \models \varphi \Rightarrow \psi & \text{iff } \mathcal{M},(t,k) \not\models \varphi \text{ or } \mathcal{M},(t,k) \models \psi \\
\mathcal{M},(t,k) \models \varphi \vee \psi & \text{iff } \mathcal{M},(t,k) \models \varphi \text{ or } \mathcal{M},(t,k) \models \psi \\
\mathcal{M},(t,k) \models \bigcirc\varphi & \text{iff } \mathcal{M},(t,k{+}1) \models \varphi \\
\mathcal{M},(t,k) \models \Box\varphi & \text{iff for all } n \in \mathbb{N},\ n \geq k \text{ implies } \mathcal{M},(t,n) \models \varphi \\
\mathcal{M},(t,k) \models \Diamond\varphi & \text{iff there exists } n \in \mathbb{N} \text{ such that } n \geq k \text{ and } \mathcal{M},(t,n) \models \varphi \\
\mathcal{M},(t,k) \models \varphi\mathcal{U}\psi & \text{iff there exists } n \in \mathbb{N} \text{ such that } n \geq k,\ \mathcal{M},(t,n) \models \psi, \text{ and} \\
 & \quad \text{for all } m \in \mathbb{N},\ k \leq m < n \text{ implies } \mathcal{M},(t,m) \models \varphi \\
\mathcal{M},(t,k) \models \varphi\mathcal{W}\psi & \text{iff } \mathcal{M},(t,k) \models \varphi\mathcal{U}\psi \text{ or } \mathcal{M},(t,k) \models \Box\varphi \\
\mathcal{M},(t,k) \models [\kappa]\,\varphi & \text{iff for all } t' \in \mathcal{T} \text{ and for all } k' \in \mathbb{N},\ ((t,k),(t',k')) \in R_\kappa \\
 & \quad \text{implies } \mathcal{M},(t',k') \models \varphi
\end{array}
$$

If $\mathcal{M}, w \models \varphi$ then we say φ is *true* or *holds* at w in $\mathcal{M}$. An interpretation $\mathcal{M}$ *satisfies* a formula φ iff φ holds at $(t_0, 0)$ and it *satisfies* a set N of formula iff for every formula $\psi \in N$, $\mathcal{M}$ satisfies ψ. In this case $\mathcal{M}$ is a *model* for φ and N, respectively.

Let $\mathcal{M}$ be an interpretation and let T be a relation on points such that $((t, k), (t', k') \in T$ iff $t = t'$ and $k' = k{+}1$. Let v, w be points in $\mathcal{M}$. Then w is *reachable* from v iff $(v, w) \in (T \cup \bigcup_\kappa R_\kappa)^*$. An interpretation $\mathcal{M}$ such that every point in $\mathcal{M}$ is reachable from $(t_0, 0)$ is a *connected interpretation*.

Note that the semantics of MTL is a notational variation of the standard (Kripke) semantics of a multi-modal logics with points corresponding to worlds. For every modal parameter κ we have a relation R_κ on worlds. In addition, there is a *temporal relation* relation T on worlds defined as the union of a family of disjoint, discrete, linear orders on the set of worlds. The semantics of $\bigcirc\varphi$ is given in terms of T while the semantics of the remaining temporal operators is given in terms of the reflexive, transitive closure T^* of T. In case we have associated additional axiom schemata to a modal operator $[\kappa]$, the relation R_κ has to satisfy the well-known corresponding properties, that is, transitivity for $\mathsf{4}$, euclideanness for $\mathsf{5}$, symmetry for B, seriality for D, and reflexivity for T.

3 A normal form for MTL formulae

Dixon et al. (1998) have shown that every well-formed formulae of MTL can be transformed to a set of *SNF*$_K$ *clauses* in a satisfiability equivalence preserving way. The use of SNF$_K$ clauses eases the presentation of a resolution calculus for MTL as well as the soundness, completeness and termination proof.

The transformation Π_K to SNF$_K$ clauses uses a renaming technique where particular subformulae are replaced by new propositions. To ease the presentation of SNF$_K$ clauses we use the universal modality $\Box^*$ as an auxiliary modal operator. The modal operator $\Box^*$ has the following important property.

Theorem 1 (Goranko and Passy 1992). *Let $\mathcal{M}$ be an interpretation, let v, w be points in $\mathcal{M}$, and let φ be a formula such that $\mathcal{M}, v \models \Box^*\varphi$. Then $\mathcal{M}, w \models \varphi$ iff w is reachable from v.*

In connected interpretations also the following stronger result holds.

Theorem 2. *Let $\mathcal{M}$ be a connected interpretation and φ be a well-formed formula of* MTL*. Then $\mathcal{M}, (t_0, 0) \models \Box^*\varphi$ iff $\mathcal{M}, w \models \varphi$ for every point w in $\mathcal{M}$.*

SNF$_K$ clauses have the following form

(initial clause)	$\mathsf{start} \Rightarrow \bigvee_{i=1}^{n} L_i$
(global clause)	$\Box^*(\bigwedge_{j=1}^{m} K_j \Rightarrow \bigcirc(\bigvee_{i=1}^{n} L_i))$
(sometime clause)	$\Box^*(\bigwedge_{j=1}^{m} K_j \Rightarrow \Diamond L)$
(literal clause)	$\Box^*(\mathsf{true} \Rightarrow \bigvee_{i=1}^{n} L_i)$
(modal clause)	$\Box^*(\mathsf{true} \Rightarrow L_1 \vee M_1)$

where K_j, L_i, and L (with $1 \leq j \leq m$ and $1 \leq i \leq n$) are literals and M_1 is a modal literal.

We only present the part of Π_K dealing with formulae of the form $\Box\varphi$ and $\varphi\mathcal{W}\psi$ which is important for the understanding of the temporal resolution rule and the example derivation presented later. For a complete description of Π_K see Dixon et al. (1998).

$$\{A \Rightarrow \Box\varphi\} \rightarrow \left\{\begin{array}{l} A \Rightarrow B \\ B \Rightarrow \bigcirc B \\ B \Rightarrow \varphi \end{array}\right\} \qquad \text{if } \varphi \text{ is a literal and } B \text{ is new}$$

$$\{A \Rightarrow \varphi\mathcal{W}\psi\} \rightarrow \left\{\begin{array}{l} A \Rightarrow \varphi \vee \psi \\ A \Rightarrow B \vee \psi \\ B \Rightarrow \bigcirc(\varphi \vee \psi) \\ B \Rightarrow \bigcirc(B \vee \psi) \end{array}\right\} \text{if } \varphi \text{ and } \psi \text{ are literals and } B \text{ is new}$$

Theorem 3 (Dixon et al. 1998). *Let φ be a formula of* MTL*. Then φ is satisfiable if and only if $\Pi_K(\varphi)$ is satisfiable in a connected interpretation.*

4 Translation of $\mathbf{SNF}_K$ clauses into $\mathbf{SNF}_r$ clauses

In the approach of Dixon et al. (1998) the calculus for MTL consists of a set of special resolution inference rules for SNF_K clauses. Broadly, these rules can be divided into two classes: those dealing with SNF_K clauses containing temporal operators and those dealing with SNF_K clauses containing modal operators. Inference rules in the later class also take care that the calculus is complete if we have associated the axiom schemata of $\mathsf{S5}$ with all modal operators.

Instead of using additional resolution rules for modal literals we use the translation approach to modal theorem proving. That is, we translate occurrences of modal literals into first-order logic, in particular, we do so in a way that preserves satisfiability and makes the use of first-order resolution possible. However, the temporal connectives are not translated and we will include additional inference rules for them in our calculus. Intuitively, the proposed translation makes the underlying relations R_κ on points explicit as well as the quantificational effect of the modal operator $[\kappa]$ explicit in our language, but still leaves the relations and quantificational effect of the temporal operators implicit.

The translation function π_r on literals, and conjunctions and disjunctions of literals is defined as follows.

$$\begin{aligned}
\pi_r(\mathsf{true}, x) &= \mathsf{true} & \pi_r(\Diamond\varphi, x) &= \Diamond\pi_r(\varphi, x)\\
\pi_r(\mathsf{false}, x) &= \mathsf{false} & \pi_r(\bigcirc\varphi, x) &= \bigcirc\pi_r(\varphi, x)\\
\pi_r(p, x) &= q_p(x) & \pi_r([\kappa]\, L, x) &= \forall y\, (\neg r_\kappa(x, y) \vee \pi_r(L, y))\\
\pi_r(\neg p, x) &= \neg q_p(x) & \pi_r(\neg\, [\kappa]\, L, x) &= r_\kappa(x, f(x)) \wedge \pi_r({\sim} L, f(x))\\
\pi_r(\varphi \star \psi, x) &= \pi_r(\varphi, x) \star \pi_r(\psi, x) & \text{for } \star &\in \{\wedge, \vee, \Rightarrow\}
\end{aligned}$$

p is a propositional variable, q_p is a unary predicate symbol uniquely associated with p, L is a literal, and f is a Skolem function uniquely associated with an occurrence of $[\kappa]\, L$. In addition, following de Nivelle (1999) the mapping τ_r on modal literals is defined by

$$\tau_r([\kappa]\, L, x) = q_{[\kappa]\, L}(x) \qquad\qquad \tau_r(\neg\, [\kappa]\, L, x) = \neg q_{[\kappa]\, L}(x)$$

where $q_{[\kappa]\, L}$ is a new predicate symbol uniquely associated with $[\kappa]\, L$.

The translation Π_r on SNF_K clauses is defined in the following way:

$$\begin{aligned}
\Pi_r(\mathsf{start} \Rightarrow \textstyle\bigvee_{i=1}^n L_i) &= \{\pi_r(\mathsf{true} \Rightarrow \textstyle\bigvee_{i=1}^n L_i, \mathsf{now})\}\\
\Pi_r(\Box^*(\textstyle\bigwedge_{j=1}^m K_j \Rightarrow \bigcirc(\bigvee_{i=1}^n L_i))) &= \{\forall x\, \pi_r(\textstyle\bigwedge_{j=1}^m K_j \Rightarrow \bigcirc(\bigvee_{i=1}^n L_i), x)\}\\
\Pi_r(\Box^*(\textstyle\bigwedge_{j=1}^m L_j \Rightarrow \Diamond L)) &= \{\forall x\, \pi_r(\textstyle\bigwedge_{j=1}^m K_j \Rightarrow \Diamond L, x)\}\\
\Pi_r(\Box^*(\mathsf{true} \Rightarrow \textstyle\bigvee_{i=1}^n L_i)) &= \{\forall x\, \pi_r(\mathsf{true} \Rightarrow \textstyle\bigvee_{i=1}^n L_i, x)\}\\
\Pi_r(\Box^*(\mathsf{true} \Rightarrow L_1 \vee M_1)) &= \left\{\begin{array}{l}\forall x\, (\mathsf{true} \Rightarrow \pi_r(L_1, x) \vee \tau_r(M_1, x))\\ \forall x\, (\mathsf{true} \Rightarrow {\sim}\tau_r(M_1, x) \vee \pi_r(M_1, x))\end{array}\right\}
\end{aligned}$$

The translation of a set N of SNF_K clauses is given by

$$\Pi_r(N) = \bigcup_{C \in N} \Pi_r(C).$$

4	Transitivity	$\forall x, y\,(\mathsf{true} \Rightarrow \neg q_{[\kappa]\,L}(x) \lor \neg r_\kappa(x,y) \lor q_{[\kappa]\,L}(y))$
5	Euclideanness	$\forall x, y\,(\mathsf{true} \Rightarrow \neg q_{[\kappa]\,L}(y) \lor \neg r_\kappa(x,y) \lor q_{[\kappa]\,L}(x))$
B	Symmetry	$\forall x, y\,(\mathsf{true} \Rightarrow \neg q_{[\kappa]\,L}(y) \lor \neg r_\kappa(x,y) \lor \pi_r(L,x))$
D	Seriality	$\forall x\,(\mathsf{true} \Rightarrow r_\kappa(x, f(x)))$
T	Reflexivity	$\forall x\,(\mathsf{true} \Rightarrow r_\kappa(x,x))$

Table 1. Translation of axiom schemata

The formulae obtained by applying Π_r to SNF_K clauses will be called SNF_r clauses. The target language of Π_r can be viewed as a fragment of first-order logic allowing only unary and binary predicate symbols extended by the the temporal operators $\bigcirc$ and $\Diamond$ or as a fragment of first-order temporal logic with the same restriction on predicate symbols and temporal operators. However, the semantics of the target language does not coincide with either of these as we will see below. In Section 5 we present a syntactic characterisation of the class of SNF_r clauses. The universal quantifiers in a SNF_r clause are usually omitted in our presentation. Any free variable in a SNF_r clause is assumed to be implicitly universally quantified.

Again, following de Nivelle (1999), depending on the additional properties of a modal operator $[\kappa]$ SNF_r clauses from Table 1 are added to the set of SNF_r clauses for every predicate symbol $q_{[\kappa]\,L}$ introduced by Π_r. The semantics of SNF_r clauses is given by temporal interpretations. A *temporal interpretation* is a tuple $(\mathcal{M}_r, \mathcal{I})$ where $\mathcal{M}_r$ is a tuple $(\mathcal{T}, t_0, \iota)$ such that $\mathcal{T}$ is a set of timelines with a distinguished timeline $t_0 \in \mathcal{T}$, ι is a morphism mapping n-ary predicate symbols to n-ary relations on $\mathcal{P}$, and $\mathcal{I}$ is a *interpretation function* mapping the constant now to $(t_0, 0)$, every variable symbol x to an element of $\mathcal{P}$, and every unary Skolem function f to a morphism $\mathcal{I}(f) : \mathcal{P} \to \mathcal{P}$. The function $\mathcal{I}$ is extended to a function $\cdot^{\mathcal{I}}$ mapping terms to $\mathcal{P}$ in the standard way, that is, $t^{\mathcal{I}} = \mathcal{I}(t)$ if t is a variable or constant, and $f(t_1, \ldots, t_n)^{\mathcal{I}} = \mathcal{I}(f)(t_1^{\mathcal{I}}, \ldots, t_n^{\mathcal{I}})$, otherwise.

Let $\mathcal{I}$ be an interpretation function. By $\mathcal{I}[x/w]$, where x is a variable and w is a point, we denote a interpretation function $\mathcal{I}'$ such that $\mathcal{I}'(y) = \mathcal{I}(y)$ for any symbol y distinct from x, and $\mathcal{I}'(x) = w$. If $x_1, \ldots, x_n$ are distinct variables and $w_1, \ldots, w_n$ are points, then $\mathcal{I}[x_1/w_1, \ldots, x_n/w_n]$ denotes $\mathcal{I}[x_1/w_1] \ldots [x_n/w_n]$. If $w = (t, k)$ is a point and $n \in \mathbb{N}$, then w^{+n} denotes the point $(t, k+n)$. If f is a mapping from points to points, then f^{+n} denotes a function defined by $f^{+n}(w) = f(w)^{+n}$ for every $w \in \mathcal{P}$, that is, for every $w \in \mathcal{P}$, if $f(w) = (t, k)$, then $f^{+n}(w) = (t, k+n)$. By $\mathcal{I}^{+n}$ we denote a interpretation function defined by $\mathcal{I}^{+n}(s) = \mathcal{I}(s)^{+n}$ for every symbol s in the domain of $\mathcal{I}$.

$(\mathcal{M}_r, \mathcal{I}) \models \mathsf{true}$

$(\mathcal{M}_r, \mathcal{I}) \not\models \mathsf{false}$

$(\mathcal{M}_r, \mathcal{I}) \models p(t_1, \ldots, t_n)$ iff $(t_1^{\mathcal{I}}, \ldots, t_n^{\mathcal{I}}) \in \iota(p)$

$(\mathcal{M}_r, \mathcal{I}) \models \neg\varphi$ iff $(\mathcal{M}_r, \mathcal{I}) \not\models \varphi$

$$
\begin{array}{ll}
(\mathcal{M}_r, \mathcal{I}) \models \varphi \wedge \psi & \text{iff } (\mathcal{M}_r, \mathcal{I}) \models \varphi \text{ and } (\mathcal{M}_r, \mathcal{I}) \models \psi \\
(\mathcal{M}_r, \mathcal{I}) \models \varphi \vee \psi & \text{iff } (\mathcal{M}_r, \mathcal{I}) \models \varphi \text{ or } (\mathcal{M}_r, \mathcal{I}) \models \psi \\
(\mathcal{M}_r, \mathcal{I}) \models \varphi \Rightarrow \psi & \text{iff } (\mathcal{M}_r, \mathcal{I}) \not\models \varphi \text{ or } (\mathcal{M}_r, \mathcal{I}) \models \psi \\
(\mathcal{M}_r, \mathcal{I}) \models \bigcirc\varphi & \text{iff } (\mathcal{M}_r, \mathcal{I}^{+1}) \models \varphi \\
(\mathcal{M}_r, \mathcal{I}) \models \Diamond\varphi & \text{iff there exists } n \in \mathbb{N} \text{ such that } (\mathcal{M}_r, \mathcal{I}^{+n}) \models \varphi \\
(\mathcal{M}_r, \mathcal{I}) \models \forall x\, \varphi & \text{iff for every } w \in \mathcal{P},\ (\mathcal{M}_r, \mathcal{I}[x/w]) \models \varphi.
\end{array}
$$

If $(\mathcal{M}_r, \mathcal{I}) \models \varphi$, then $(\mathcal{M}_r, \mathcal{I})$ *satisfies* φ and φ is *satisfiable*. Although the syntax of SNF_r clauses resembles that of first-order temporal logic, the semantics is different. Unlike in first-order temporal logic variables are not interpreted as elements of domains attached to points, but are interpreted as points. Likewise constants and function symbols are not interpreted as morphisms on domains but as morphisms on points. In fact, the semantics is still based on the same building blocks: timelines, points, relations on points and mappings between points and symbols of our language (or vice versa).

However, note that for formulae not containing any occurrences of a $\bigcirc$ and $\Diamond$ operators, temporal interpretations act like standard first-order interpretations.

Theorem 4. *Let N be a set of SNF_K clauses. Then N is satisfiable if and only if $\Pi_r(N)$ is satisfiable.*

Proof. For an arbitrary connected model $\mathcal{M}$ for N we are able to construct a temporal interpretation $(\mathcal{M}_r, \mathcal{I})$ for $\Pi_r(N)$ and vice versa. The proof that $\mathcal{M}$ and $(\mathcal{M}_r, \mathcal{I})$ satisfy N and $\Pi_r(N)$, respectively, proceeds by induction on the structure of formula in N and $\Pi_r(N)$.

5 A calculus for MTL

We call a literal L *shallow* if all its argument terms are either variables or constants, otherwise it is deep. If C is a disjunction or conjunction of shallow, unary literals, then by $C[x]$ we indicate that all literals in C have a common variable argument term x, and $C[x/y]$ is obtained by replacing every occurrence of x in C by y. Similarly, if L is a monadic literal, we write $L[t]$ to indicate that the term t is the argument of L.

A *clause* is a formula of the form $P \Rightarrow \varphi$ where P is a conjunction of literals and φ is a disjunction of literals, a formula of the form $\Diamond L$, or a formula $\bigcirc C$ where C is disjunction of literals. The *empty clause* is false or $\mathsf{true} \Rightarrow \mathsf{false}$. We regard the logical connectives $\wedge$ and $\vee$ in clauses to be associative, commutative, and idempotent. Equality of clauses is taken to be equality modulo variable renaming.

A clause C is a *simple clause* if and only if all literals in C are shallow, unary, and share the same argument term. A conjunction (disjunction) C is a *simple conjunction (disjunction)* iff C is a conjunction (disjunction) of shallow, unary literals that share the same argument term. $\sim P$ denotes the negation normal form of a conjunction P.

A clause C is a *temporal clause* if and only if it either has the form $P[t] \Rightarrow \bigcirc D[t]$ or $P[x] \Rightarrow \Diamond L[x]$ where $P[x]$ is a simple conjunction, $D[t]$ is a simple clause and t is either a variable or the constant now. A clause C is a *modal clause* iff it either has the form $P[x] \Rightarrow r_\kappa(x, f(x))$, $P[x] \Rightarrow D[f(x)]$, or $P[x] \Rightarrow \neg r_\kappa(x,y) \vee D[y]$ where P is a simple conjunction and $D[f(x)]$ is a clause of unary, deep literals with common argument $f(x)$. For a simple or modal clause C we do not distinguish between $P \Rightarrow D$ and $\mathsf{true} \Rightarrow {\sim}P \vee D$.

For every ground atom A, let the complexity measure $c(A)$ be the multiset of arguments of A. We compare complexity measures by the multiset extension $\succ_m$ of the strict subterm ordering. The ordering is lifted from ground to non-ground expressions as follows: $A \succ' B$ if and only if $c(A\sigma) \succ_m c(B\sigma)$, for all ground instances $A\sigma$ and $B\sigma$ of atoms A and B. The ordering $\succ'$ on atoms can be lifted to literals by associating with every positive literal A the multiset $\{A\}$ and with every negative literal $\neg A$ the multiset $\{A, A\}$, and comparing these by the multiset extension of $\succ'$. We denote the resulting ordering by $\succ$. The ordering $\succ$ is an admissible ordering in the sense of Bachmair and Ganzinger (1997). Thus, the following two inference rules provide a sound and complete calculus for first-order logic in clausal form:

$$\text{Res} \qquad \frac{\mathsf{true} \Rightarrow C \vee A \qquad \mathsf{true} \Rightarrow D \vee \neg B}{\mathsf{true} \Rightarrow (C \vee D)\sigma}$$

where (i) $C \vee A$ and $D \vee \neg B$ are simple or modal clauses, (ii) σ is the most general unifier of A and B, (iii) $A\sigma$ is strictly $\succ$-maximal with respect to $C\sigma$, and (iv) $\neg B\sigma$ is $\succ$-maximal with respect to $D\sigma$. As usual we assume that premises of resolution inference steps are variable disjoint.

$$\text{Fac} \qquad \frac{\mathsf{true} \Rightarrow C \vee L_1 \vee L_2}{\mathsf{true} \Rightarrow (C \vee L_1)\sigma}$$

where (i) $C \vee L_1 \vee L_2$ is a simple or modal clause, (ii) σ is the most general unifier of L_1 and L_2, and (iii) $L_1\sigma$ is $\succ$-maximal with respect to $C\sigma$.

Lemma 10 will show that the factoring inference rule is not required for the completeness of our calculus if we assume that the logical connective $\vee$ is idempotent.

The remaining inference rules are similar to those of the calculus for linear temporal logic presented in Dixon et al. (1998). An inference by *step resolution* takes one of the following forms

$$\text{SRes1} \qquad \frac{P \Rightarrow \bigcirc(C \vee L_1) \qquad Q \Rightarrow \bigcirc(D \vee L_2)}{(P \wedge Q)\sigma \Rightarrow \bigcirc(C \vee D)\sigma}$$

$$\text{SRes2} \qquad \frac{\mathsf{true} \Rightarrow C \vee L_1 \qquad Q \Rightarrow \bigcirc(D \vee L_2)}{Q\sigma \Rightarrow \bigcirc(C \vee D)\sigma}$$

$$\text{SRes3} \qquad \frac{P \Rightarrow \bigcirc\mathsf{false}}{\mathsf{true} \Rightarrow {\sim}P}$$

where (i) P and Q are simple conjunctions, (ii) $C \vee L_1$ and $D \vee L_2$ are simple clauses, and (iii) σ is the most general unifier of L_1 and ${\sim}L_2$.

The following *merge rule* allows the formation of the conjunction of temporal clauses.

$$\text{Merge} \qquad \frac{\begin{array}{c} P_0[x_0] \Rightarrow \bigcirc C_0[x_0] \\ \dots \\ P_n[x_n] \Rightarrow \bigcirc C_n[x_n] \end{array}}{\bigwedge_{i=0}^{n} P_i[y] \Rightarrow \bigcirc \bigwedge_{i=0}^{n} C_i[y]}$$

where (i) each P_i, $1 \leq i \leq n$, is a simple conjunction, (ii) each C_i, $1 \leq i \leq n$ is a simple clause, (iii) y is a new variable. The conclusion of an inference step by the merge rule is a *merged temporal clause*. The only purpose of this rule is to ease the presentation of the following temporal resolution rule.

An inference by *temporal resolution* takes the following form

$$\text{TRes} \qquad \frac{\begin{array}{c} P_0[x_0] \Rightarrow \bigcirc G_0[x_0] \\ \dots \\ P_n[x_n] \Rightarrow \bigcirc G_n[x_n] \\ A(y) \Rightarrow \diamond L[y] \end{array}}{A(y) \Rightarrow (\bigwedge_{i=0}^{n} \neg P_i[y]) \mathcal{W} L(y)}$$

where (i) each $P_i[x_i] \Rightarrow \bigcirc G_i[x_i]$, $1 \leq i \leq n$, is a merged temporal clause, (ii) for all i, $0 \leq i \leq n$, $\forall x_i\, G_i[x_i] \Rightarrow \neg L[x_i]$ and $\forall x_i\, G_i[x_i] \Rightarrow \bigvee_{j=0}^{n} P_j[x_j/x_i]$ are provable.

The conclusion $A(y) \Rightarrow (\bigwedge_{i=0,\dots,n} \neg P_i[y]) \mathcal{W} L(y)$ of an inference by temporal resolution has to be transformed into normal form. Thus, we obtain

$$\begin{aligned}
&(1) && \mathsf{true} \Rightarrow \neg A(y) \vee L(y) \vee {\sim} P_i(y) \\
&(2) && \mathsf{true} \Rightarrow \neg A(y) \vee L(y) \vee q_L^w(y) \\
&(3) && q_L^w(y) \Rightarrow \bigcirc(L(y) \vee {\sim} P_i(y)) \\
&(4) && q_L^w(y) \Rightarrow \bigcirc(L(y) \vee q_L^w(y)),
\end{aligned}$$

where q_L^w is a new unary predicate symbol uniquely associated with L.

The calculus $\mathsf{C_{MTL}}$ consists of the inference rules Res, Fac, SRes1, SRes2, SRes3, Merge, and TRes. It is possible to replace Merge and TRes by a single inference rule which uses ordinary temporal clauses as premises and forms the merged clauses only in an intermediate step to compute the conclusion of an application of the temporal resolution rule. Thus, in our consideration in Section 6 and 7 we will not explicitly mention the Merge inference rule and merged temporal clauses.

6 Soundness of $\mathsf{C_{MTL}}$

Lemma 5. *Let* $\mathsf{true} \Rightarrow C$ *be a clause and let* σ *be a substitution. If* $\mathsf{true} \Rightarrow C$ *is satisfiable, then* $\mathsf{true} \Rightarrow C\sigma$ *is satisfiable.*

Theorem 6 (Soundness). *Let* φ *be a well-formed formula of* MTL*. If a refutation of* $\Pi_r(\Pi_K(\varphi))$ *in* $\mathsf{C_{MTL}}$ *exists then* φ *is unsatisfiable.*

Proof. We show that for every instance of an inference rule of $\mathsf{C_{MTL}}$ that the satisfiability of the premises implies the satisfiability of the conclusion. In the case of Res and Fac this is straightforward since temporal interpretations act like first-order interpretations and we know that Res and Fac are sound with respect to first-order logic. The proof for the remaining inference rules can be found in (Hustadt et al. 2000).

7 Termination of $\mathsf{C_{MTL}}$

The termination proof for our calculus will take advantage of the following observations.

1. SNF_r clauses can be divided into three disjoint classes: *temporal clauses*, *modal clauses*, and *simple clauses*. It is straightforward to check that if N is a set of SNF_K clauses then all clauses in $\Pi_r(N)$ including the clauses we add for one of the axiom schemata $\mathsf{4}$, $\mathsf{5}$, B, D, and T belong to exactly one of these classes.
   Note that simple and modal clauses are standard first-order clauses.
2. The inference rules of our calculus can be divided into two classes: Res and Fac are the standard inference rules for ordered resolution and only modal clauses and simple clauses are premises in inference steps by these rules. The conclusion of such an inference step will again be a clause belonging to one of these two classes as we show in Lemma 9 and Lemma 10 below.
   SRes1, SRes2, SRes3, and TRes are variants of the inference rules of the resolution calculus for linear temporal logic presented in Dixon et al. (1998). Only temporal and simple clauses can be premises of inference steps by these rules. The conclusion of such an inference step will consist of clauses which again belong to one these classes as is shown in Lemma 11 and Lemma 12 below.

Thus, the clauses under consideration and the calculus enjoy a certain modularity. Interaction between the two classes of inference rules and the class of temporal and modal clauses are only possible via the class of simple clauses. Given a finite signature, the classes of simple, modal, and temporal clauses are finitely bounded. Termination of any derivation from SNF_r is a direct consequence of the closure properties of the inference rules mentioned above.

Lemma 7. *Let φ be a well-formed formula of* MTL. *Every clause in $\Pi_r(\Pi_K(\varphi))$ is either a simple, a temporal, or a modal clause.*

Lemma 8. *Given a finite signature Σ, the classes of simple, modal and temporal clauses over Σ are finitely bounded.*

Proof. Note that the only terms which can occur in these clauses are either variables, the constant now, or terms of the from $f(t)$ where t is either a variable or a constant. Furthermore, no clause has more than two variables. Given that we can show that the length of clauses is linear in the size of the signature, limiting the number of non-variant clauses to an exponential number in the size of the signature.

The proof shows that SNF_r clauses have a linear length in the size of the signature. That gives us a single-exponential space (and time) bound for our decision procedure.

Due to side condition (i) of the inference rules Res and Fac, temporal clauses cannot be premises of inference steps by these rules. Simple clauses and modal clauses are special cases of *DL-clauses* (Hustadt and Schmidt 2000). The following two lemmata follow directly from the corresponding result for DL-clauses.

Lemma 9. *Let $C_1 \lor A$ and $C_2 \lor \neg B$ be SNF_r clauses and let $C = (C_1 \lor C_2)\sigma$ be an ordered resolvent of these clauses. Then C is either a modal clause or a simple clause.*

Lemma 10. *Let $C_1 = D_1 \lor L_1 \lor L_2$ be a SNF_r clause and let $C = (D_1 \lor L_1)\sigma$ be a factor of C_1. Then C is either a modal clause or a simple clause and an application of the factoring rule simply amounts to the removal of duplicate literals in C_1.*

By a case analysis of all possible inference steps by SRes1, SRes2, SRes3, and TRes on SNF_r clauses we obtain the following two lemmata.

Lemma 11. *Let C_1 and C_2 be SNF_r clauses and let C be the conclusion of inference steps by SRes1, SRes2, or SRes3 from C_1 and C_2. Then C is a temporal clause or a simple clause.*

Lemma 12. *Let $C_1, \ldots, C_n$ be SNF_r clauses and let C be one of the clauses resulting from the transformation of the conclusion of an application of TRes to $C_1, \ldots, C_n$. Then C is a simple or a temporal clause.*

We are now in the position to state the main theorem of this section.

Theorem 13 (Termination). *Let φ be a well-formed formula of* MTL. *Any derivation from $\Pi_r(\Pi_K(\varphi))$ in $\mathsf{C}_{\mathsf{MTL}}$ terminates.*

Proof. By induction on the length of the derivation from $\Pi_r(\Pi_K(\varphi))$ we can show that any clause occurring in the derivation is either a simple, modal, or temporal clause. Lemma 7 proves the base case that every clauses in $\Pi_r(\Pi_K(\varphi))$ satisfies this property. Lemmata 9, 10, 11, and 12 establish the induction step of the proof.

The signature of clauses in $\Pi_r(\Pi_K(\varphi))$ is obviously finite. By Lemma 8 the classes of simple, modal, and temporal clauses based on a finite signature is finitely bounded. Thus, after a finitely bounded number of inference step we will have derived the empty clause or no new clauses will be added to the set of clauses. In both cases the derivation terminates.

8 Completeness of $\mathsf{C}_{\mathsf{MTL}}$

The proof of completeness proceeds as follows. We describe a canonical construction of a *behaviour graph* and *reduced behaviour graph* for a given set N

of SNF_r clauses. In Theorem 14 we show that N is unsatisfiable if and only if its reduced behaviour graph is empty. Theorem 16 shows that if the reduced behaviour graph for N is empty, then we are able to derive a contradiction using $\mathsf{C}_{\mathsf{MTL}}$. Theorem 14 and 16 together imply that for any unsatisfiable set N of SNF_r clauses we can derive a contradiction. Thus, $\mathsf{C}_{\mathsf{MTL}}$ is complete. Details of the construction of behaviour graphs, reduced behaviour graphs, and the proof for the results of this section can be found in (Hustadt et al. 2000).

Theorem 14. *Let N be a set of SNF_r clauses. Then N is unsatisfiable if and only if its reduced behaviour graph is empty.*

Proof. The constructions are similar to those in Dixon et al. (1998), except that we have explicit nodes and edges for the modal dimension for our logic, which were not necessary in the case that we have only the modal logic $\mathsf{S5}$.

Lemma 15. *Let N be a set of SNF_r clauses. If the unreduced behaviour graph for N is empty, then we can derive a contradiction from N using only the inference rules Res, SRes1, SRes2, and SRes3 of* $\mathsf{C}_{\mathsf{MTL}}$.

Proof. If the unreduced behaviour graph is empty, then any node we have constructed originally, has been deleted because one of the simple, modal, or temporal clauses of the form $P[x] \Rightarrow \bigcirc C[x]$ is not true at n_s. Thus, we can use the inference rules Res, SRes1, SRes2, and SRes3 to derive a contradiction.

Theorem 16. *Let N be a set of SNF_r clauses. If the reduced behaviour graph for N is empty, then we can derive a contradiction from N by* $\mathsf{C}_{\mathsf{MTL}}$.

Proof. Let N be an unsatisfiable set of SNF_r rules. The proof is by induction on the number of nodes in the behaviour graph of N. If the unreduced behaviour graph is empty, then by Lemma 15 we can obtain a refutation using the inference rules Res, SRes1, SRes2, and SRes3.

Suppose that the unreduced behaviour graph G is non-empty. By Theorem 14 the reduced behaviour graph must be empty, so each node in G can be deleted by reduction rules similar to those in (Dixon et al. 1998). The deletion of these nodes are shown to correspond to applications of step resolution and temporal resolution along the lines of Dixon et al. (1998).

The completeness theorem now follows from Theorems 3, 4, and 16.

Theorem 17 (Completeness). *Let φ be a well-formed formula of* MTL. *If φ is unsatisfiable, then there exists a refutation of $\Pi_r(\Pi_K(\varphi))$ by* $\mathsf{C}_{\mathsf{MTL}}$.

9 Example refutation

We show that $[K]\bigcirc p \wedge \Box [K](p \Rightarrow \bigcirc p) \Rightarrow \bigcirc\Box p$ is valid if $[K]$ is a T modality. This is done by proving the unsatisfiability of

$$\varphi = [K]\bigcirc p \wedge \Box [K](p \Rightarrow \bigcirc p) \wedge \bigcirc\Diamond\neg p$$

$\Pi_r(\Pi_K(\varphi))$ is equal to the following set of clauses:

(5)	$\mathsf{true} \Rightarrow q_0(\mathsf{now})$
(6)	$\mathsf{true} \Rightarrow \neg q_0(x) \vee q_{[K]\bigcirc p}(x)$
(7)	$\mathsf{true} \Rightarrow \neg q_0(x) \vee q_1(x)$
(8)	$\mathsf{true} \Rightarrow \neg q_0(x) \vee q_2(x)$
(9)	$\mathsf{true} \Rightarrow \neg q_{[K]\bigcirc p}(x) \vee \neg r_K(x,y) \vee q_3(y)$
(10)	$q_3(z) \Rightarrow \bigcirc q_p(z)$
(11)	$\mathsf{true} \Rightarrow \neg q_1(x) \vee q_{[K](p\Rightarrow\bigcirc p)}(x)$
(12)	$q_1(x) \Rightarrow \bigcirc q_1(x)$
(13)	$\mathsf{true} \Rightarrow \neg q_{[K](p\Rightarrow\bigcirc p)}(x) \vee \neg r_K(x,y) \vee q_4(y)$
(14)	$\mathsf{true} \Rightarrow \neg q_4(x) \vee \neg q_p(x) \vee q_5(x)$
(15)	$q_5(x) \Rightarrow \bigcirc q_p(x)$
(16)	$q_2(x) \Rightarrow \bigcirc q_6(x)$
(17)	$q_6(x) \Rightarrow \Diamond \neg q_p(x)$
(18)	$r_K(x,x)$

The derivation proceeds as follows:

[(18)1,(13)2,Res]	(19)	$\mathsf{true} \Rightarrow \neg q_{[K](p\Rightarrow\bigcirc p)}(x) \vee q_4(x)$
[(11)1,(12)2,SRes2]	(20)	$q_1(x) \Rightarrow \bigcirc q_{[K](p\Rightarrow\bigcirc p)}(x)$
[(19)1,(20)2,SRes2]	(21)	$q_1(x) \Rightarrow \bigcirc q_4(x)$
[(14)1,(21)2,SRes2]	(22)	$q_1(x) \Rightarrow \bigcirc(\neg q_p(x) \vee q_5(x))$
[(15)2,(22)2,SRes1]	(23)	$q_1(x) \wedge q_5(x) \Rightarrow \bigcirc q_5(x)$
[(12),(15),(23),Merge]	(24)	$q_1(x) \wedge q_5(x) \Rightarrow \bigcirc(q_1(x) \wedge q_5(x) \wedge q_p(x))$

Intuitively clause (24) says that once q_1 and q_5 hold at a point x, q_1, q_5, and q_p will hold at any temporal successor of x. Thus, once we reach x we will not be able to satisfy $\Diamond\neg q_p(x)$. This gives rise to an application of the temporal resolution rule to (17) and (24). We obtain the following four clauses from the conclusion of this inference step.

[(17),(24),TRes]	(25)	$\mathsf{true} \Rightarrow \neg q_6(x) \vee \neg q_p(x) \vee \neg q_1(x) \vee \neg q_5(x)$
	(26)	$\mathsf{true} \Rightarrow \neg q_6(x) \vee \neg q_p(x) \vee q_7(x)$
	(27)	$q_7(x) \Rightarrow \bigcirc(\neg q_p(x) \vee \neg q_1(x) \vee \neg q_5(x))$
	(28)	$q_7(x) \Rightarrow \bigcirc(\neg q_p(x) \vee q_7(x))$

In the following only clause (25) will be relevant. We show now that q_1, q_2 and q_3 cannot be true at the same point x.

[(14)3,(25)4,Res]	(29)	$\mathsf{true} \Rightarrow \neg q_6(x) \vee \neg q_p(x) \vee \neg q_1(x) \vee \neg q_4(x)$
[(19)2,(29)4,Res]	(30)	$\mathsf{true} \Rightarrow \neg q_6(x) \vee \neg q_p(x) \vee \neg q_1(x) \vee \neg q_{[K](p\Rightarrow\bigcirc p)}(x)$
[(11)2,(30)4,Res]	(31)	$\mathsf{true} \Rightarrow \neg q_6(x) \vee \neg q_p(x) \vee \neg q_1(x)$
[(12)2,(31)3,SRes2]	(32)	$q_1(x) \Rightarrow \bigcirc(\neg q_6(x) \vee \neg q_p(x))$
[(16)2,(32)3,SRes1]	(33)	$q_1(x) \wedge q_2(x) \Rightarrow \bigcirc\neg q_p(x)$
[(10)2,(33)3,SRes1]	(34)	$q_1(x) \wedge q_2(x) \wedge q_3(x) \Rightarrow \bigcirc\mathsf{false}$
[(34),SRes3]	(35)	$\mathsf{true} \Rightarrow \neg q_1(x) \vee \neg q_2(x) \vee \neg q_3(x)$

The remainder of the refutation is straightforward. Based on the clauses (5) to (9) and the reflexivity of r_K, it is easy to see that q_1, q_2, and q_3 are true at the point now which contradicts clause (35).

[(18)1,(9)2,Res]	(36)	$\mathsf{true} \Rightarrow \neg q_{[K]\bigcirc p}(x) \vee q_3(x)$
[(36)2,(35)3,Res]	(37)	$\mathsf{true} \Rightarrow \neg q_1(x) \vee \neg q_2(x) \vee \neg q_{[K]\bigcirc p}(x)$
[(6)2,(37)3,Res]	(38)	$\mathsf{true} \Rightarrow \neg q_1(x) \vee \neg q_2(x) \vee \neg q_0(x)$
[(7)2,(38)1,Res]	(39)	$\mathsf{true} \Rightarrow \neg q_2(x) \vee \neg q_0(x)$
[(8)2,(39)1,Res]	(40)	$\mathsf{true} \Rightarrow \neg q_0(x)$
[(5)1,(40)1,Res]	(41)	$\mathsf{true} \Rightarrow \mathsf{false}$

10 Conclusion

We have presented a framework for the combination of modal and temporal logics consisting of (i) a normal form transformation of formulae of the combined logics into sets of SNF_K clauses, (ii) a translation of modal subformula in SNF_K clauses into a first-order language, and (iii) a calculus $\mathsf{C}_{\mathsf{MTL}}$ for the combined logic which can be divided into standard resolution inference rules for first-order logic and modified resolution inference rules for discrete linear temporal logic.

The calculus $\mathsf{C}_{\mathsf{MTL}}$ provides a decision procedure for combinations of subsystems of multi-modal $\mathsf{S5}$ with linear, temporal logic.

Note that instead of modifying the inference rules for discrete linear temporal logic we could have retained them in their original form and added *bridging rules* between the two logics. We have shown that the only clauses which can be premises of the first-order inference rules as well as of the temporal inference rules are simple clauses. So, assume that Π_r leaves any SNF_K clauses with occurrences of the temporal connective $\bigcirc$ and $\Diamond$ unchanged. Furthermore, let δ be the homomorphic extension of a function that maps atoms $q_p(x)$ to q_p. Then an alternative calculus to $\mathsf{C}_{\mathsf{MTL}}$ consists of Res, Fac, the original step resolution rules and temporal inference rule by Dixon et al. (1998), and the two bridging rules

$$br_{pl} \quad \frac{\mathsf{true} \Rightarrow C}{\mathsf{true} \Rightarrow \delta(C)} \qquad\qquad br_{fol} \quad \frac{\mathsf{true} \Rightarrow \delta(C)}{\mathsf{true} \Rightarrow C}$$

where $\mathsf{true} \Rightarrow C$ is a simple clause. This again stresses the importance of the observation that simple clauses control the interaction between the two calculi involved. The bridging rules allow for the translation of simple clauses during the derivation, thus providing an interface between the two calculi we have combined. This is approach is closely related to the work by Ghidini and Serafini (1998).

Although we have only considered the basic modal logic K and its extensions by the axiom schemata $\mathsf{4}$, $\mathsf{5}$, B, D, and T, we are confident that soundness, completeness, and termination can be guaranteed for a much wider range of modal logics.

An important extension of the combinations of modal logics we are currently investigating is the addition of interactions between modal and temporal logics. In the presence of interactions the modularity of the calculus and the modularity of our proofs, in particular the proof of termination, can no longer be preserved.

References

Baader, F. and Ohlbach, H. J. (1995). A multi-dimensional terminological knowledge representation language. *Journal of Applied Non-Classical Logics*, 2:153–197.

Bachmair, L. and Ganzinger, H. (1997). A theory of resolution. Research report MPI-I-97-2-005, Max-Planck-Institut für Informatik, Saarbrücken, Germany. To appear in J. A. Robinson and A. Voronkov, editors, *Handbook of Automated Reasoning*.

Blackburn, P. and de Rijke, M. (1997). Why combine logics? *Studia Logica*, 59:5–27.

de Nivelle, H. (1999). Translation of S4 into GF and 2VAR. Manuscript.

Dixon, C., Fisher, M., and Wooldridge, M. (1998). Resolution for temporal logics of knowledge. *Journal of Logic and Computaton*, 8(3):345–372.

Fagin, R., Halpern, J. Y., Moses, Y., and Vardi, M. Y. (1996). *Reasoning About Knowledge*. MIT Press.

Finger, M. (1994). Notes on several methods for combining temporal logic systems. Presented at ESSLLI'94.

Gabbay, D. M. (1996). Fibred semantics and the weaving of logics. Part 1. Modal and intuitionistic logics. *Journal of Symbolic Logic*, 61(4):1057–1120.

Ghidini, C. and Serafini, L. (1998). Distributed first order logics. In Gabbay, D. M. and de Rijke, M., editors, *Proc. FroCoS'98*. To appear.

Goranko, V. and Passy, S. (1992). Using the universal modality: Gains and questions. *Journal of Logic and Computation*, 2(1):5–30.

Halpern, J. Y. (1987). Using reasoning about knowledge to analyse distributed systems. *Annual Review of Computer Science*, 2.

Hustadt, U., Dixon, C., Schmidt, R., and Fisher, M. (2000). Normal forms and proofs in combined modal and temporal logics. Extended version of this paper, available at `http://www.card.mmu.ac.uk/U.Hustadt/publications/HDSF2000b.ps.gz`.

Hustadt, U. and Schmidt, R. A. (2000). Issues of decidability for description logics in the framework of resolution. In Caferra, R. and Salzer, G., editors, *Automated Deduction in Classical and Non-Classical Logics*, volume 1761 of *LNAI*, pages 192–206. Springer.

Jennings, N. R. (1999). Agent-based computing: Promise and perils. In Dean, T., editor, *Proc. IJCAI'99*, pages 1429–1436. Morgan Kaufmann.

Johnson, C. W. (1994). The formal analysis of human-computer interaction during accidents investigations. In *People and Computers IX*, pages 285–300. Cambridge University Press.

Ohlbach, H. J. (1998). Combining Hilbert style and semantic reasoning in a resolution framework. In Kirchner, C. and Kirchner, H., editors, *Proc. CADE-15*, volume 1421 of *LNAI*, pages 205–219. Springer.

Ohlbach, H. J. and Gabbay, D. M. (1998). Calendar logic. *Journal of Applied Non-Classical Logics*, 8(4).

Wolter, F. and Zakharyaschev, M. (1998). Satisfiability problem in description logics with modal operators. In Cohn, A. G., Schubert, L. K., and Shapiro, S. C., editors, *Proc. KR'98*, pages 512–523. Morgan Kaufmann.

Wooldridge, M., Dixon, C., and Fisher, M. (1998). A tableau-based proof method for temporal logics of knowledge and belief. *Journal of Applied Non-Classical Logics*, 8(3):225–258.

Wooldridge, M. and Jennings, N. R. (1995). Intelligent agents: Theory and practice. *The Knowledge Engineering Review*, 10(2):115–152.