DAY7

64) An electric potential ϕ is given by

$$\phi(x, y, z) = xy \sin z + x^2 y + y^2 z + z^2 x$$

Find the directional derivative of the electric potential ϕ at the point $P(1, -1, \pi)$ in the direction of the vector $\boldsymbol{n} = \boldsymbol{i} - \boldsymbol{j} - \boldsymbol{k}$.

65) A total resistance Z is given by the formula

$$\frac{1}{Z} = \jmath\omega L + \frac{1}{\jmath\omega C} + \frac{1}{R}$$

Find the derivative $\frac{dZ}{dC}$.

66) Let $P = P(x, y)$, and $x = \mathfrak{e}^t$ and $y = \mathfrak{e}^{-t}$. Find the total derivative $\frac{dP}{dt}$ in terms of partial derivatives $\frac{\partial P}{\partial x}$ and $\frac{\partial P}{\partial y}$. Hence find the second total derivative $\frac{d^2 P}{dt^2}$ in terms of partial derivatives $\frac{\partial P}{\partial x}$, $\frac{\partial P}{\partial y}$, $\frac{\partial^2 P}{\partial x^2}$, $\frac{\partial^2 P}{\partial y^2}$, and $\frac{\partial^2 P}{\partial x \partial y}$. You may assume that the two mixed partial derivatives are equal.

DAY8

67) Locate all stationary points for the function $f(x,y) = 2x^3 + 3x^2y + 2y^3 - 144y + 7$. How many stationary points are there ?

68) A point $(x,y) = (-4,-8)$ is one of the stationary points of the function $f(x,y) = 12xy - 3y^2 + 2x^3$. Find the nature of this stationary point.

69) Explain why, for the function $f(x,y) = (x+y)\mathfrak{e}^{-xy}$, the stationary point at $x = \frac{1}{\sqrt{2}}$, $y = \frac{1}{\sqrt{2}}$ is a saddle point despite both $\dfrac{d^2 f(x,y)}{dx^2}$ and $\dfrac{\partial^2 f(x,y)}{\partial y^2}$ being negative.

DAY4

46) The current, I, is given by

$$I(V) = I_s \sinh(V)$$

where V is the applied voltage and I_s is a constant. If the operating voltage is given by $V_a = \pi$ (measured in Volts), find a second order Taylor approximation for $I(V)$ about this operating voltage.

47) The current, I, is given by

$$I(V, t) = \mathfrak{e}^{-V} \cos(\omega t)$$

where V is the applied voltage and t is time. Find the term in $t^2 V^3$ in the Taylor series expansion around $t = 0, V = 0$.

48) The current, I, is given by

$$I(V, R) = \frac{\mathfrak{e}^V}{R}$$

where V is the applied voltage and R is a variable.

a) Find the second order Taylor series for I around $V = 0, R = 1$

b) Using the series estimate $I(0.1, 0.9)$ and compare it with the exact value of $I(0.1, 0.9)$

DAY7

71) A loudspeaker cone is generated by rotating the curve $y = \cosh x - 1$ about the $x-$ axis through 2π radians from $x = 0$ to $x = 1$. Calculate the surface area of the cone excluding the two ends.

[5 marks]

72) For the force

$$\mathbf{F} = (y + 3x^2z^2)\mathbf{i} + (x - z)\mathbf{j} + (2x^3z - y)\mathbf{k}$$

find the potential ϕ such that $\mathbf{F} = \nabla\phi$. Hence evaluate

$$\int_{(0,0,0)}^{(1,2,3)} (y + 3x^2z^2)dx + (x - z)dy + (2x^3z - y)dz$$

73) For the double integral

$$\int_{-1}^{0}\int_{x}^{-x^2} 12xydydx$$

draw a clear, labelled sketch of the region of integration and evaluate the integral using any suitable method.

74) Find the work

$$W = \int_C \mathbf{F} \cdot d\boldsymbol{r}$$

done by the force $\mathbf{F} = x^2\boldsymbol{i} + xy\boldsymbol{j}$ in moving a particle along the curve given parametrically by

$$x(t) = 1 - t$$

and

$$y(t) = t$$

where $0 \le t \le 1$.

DAY5

65) Consider the differential equation

$$\frac{d\{I\}}{dt} - I = r(t)$$

a) For the homogeneous equation with $r(t) = 0$, find the general solution.
b) Find the general solution to the homogeneous equation charaterised by $r(t) = \mathfrak{e}^{-t}$ and the particular solution for $I = 0.5$ at $t = 0$ and describe the behavior for large t

66) Solve the differential equation

$$\frac{\partial^2 I}{\partial t^2} - I = 2\mathfrak{e}^{-t} - 1$$

subject to the conditions that I remains finite for large t and that $I = 2$ when $t = 0$

67) The current $I(t)$ at time t in an electric circuit with total resistance R and self-inductance L satisfies

$$L\frac{d\{I\}}{dt} + RI = \mathfrak{e}^{-Dt}$$

where $0 < D < \frac{R}{L}$.

a) Find the general solution to the problem.
b) The switch is closed at $t = 0$ and the initial value of the current is $I(0) = 2$. When $R = 6, L = 1, D = 5$, find the solution to this initial value problem.
c) Write down the steady-state value I_s of the current