MSc COMP6012

Automated Reasoning

Who, What, When, Where, Why?

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September 2006

Why?

System Design:

- The Pentium Bug

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- The Pentium II Bug

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- Future: Internet encryption bug???. . .

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- The Pentium Bug
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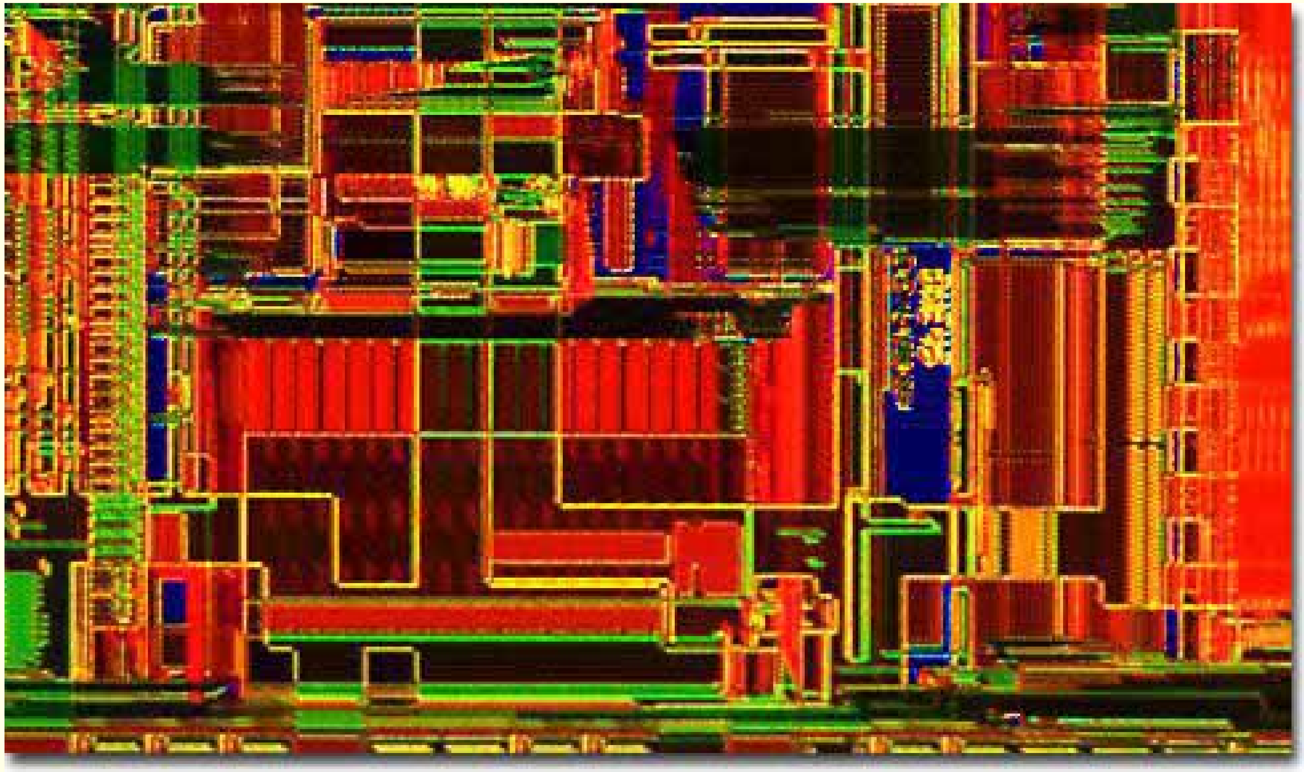
- Arriane 5 Failure, 4 June 1996
- software + hardware specification and design errors ...
- increasing design complexity ...
- Future: Internet encryption bug???...hasn't been found...yet

Or...

- Mathematical Logical Foundations

http://www.dutchspace.nl/



http://micro.magnet.fsu.edu

Why You May Wish To Take COMP6012

- Inform/support other MSc course units (but not pre/co-requisites):
    - COMP6016: Knowledge Representation and Reasoning
    - COMP6039: Computer Security
    - COMP6046: The Semantic Web: Ontologies and OWL
- Mathematical Logic
- System Design: hardware, software, GRID, secure, biological. . .
- Design tool development: CAD, IDEs

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- (System property or component description via formal logic)
- Logical reasoning
- Automation: decision procedures
- Advanced techniques for efficiency
- Associated theoretical concepts, e.g. soundness and completeness

Course Outline

When?

Period 1, Semester 1

Mondays

Where?

Lectures: 2.15

Labs: 2.25a

A Course of Two Halves:

1. Formal Logic and Automated Reasoning (AJW)
2. Advanced Automated Reasoning (RenS)

Pre-requisites: Familiarity with Propositional Logic

Part I: Formal Logic and Automated Reasoning

- Classical Propositional Logic
- First-order Predicate Logic
- Automated Reasoning: Methods and Tools, including
  - resolution
  - logic programming

Reasoning Example

Assumptions:

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IF I live in Manchester THEN it is SUNNY

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IF it is raining THEN I need a PARASOL

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Conclusion:

IF I live in Manchester THEN I need an PARASOL

The Resolution Principle

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The basis of

- Automated Theorem-proving: e.g. Vampire (Andrei Voronkov)
- Logic Programming: e.g. Prolog

Logic Programming and Prolog

Prolog Program — rules and facts:

```
ancestor(X,Y) :- parent(X,Y).

ancestor(X,Y) :- parent(X,Z),
                 ancestor(Z,Y).

parent(sue,toby).
parent(roy,sue).
```

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Run program:

```
?- ancestor(roy,X).

X = sue;
X = toby;
```

Part II: Advanced Techniques

Why:

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  - Extremely prolific at generating new conclusions
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Why:

- The basic resolution calculus is very simple
  - Just two rules
  - Extremely prolific at generating new conclusions
  - Inefficient, impracticable
- Advanced techniques are available
- Part II is devoted to Advanced Automated Reasoning

Emphasis in Part II

- Foundations of advanced automated theorem proving
  - Selection of important topics
  - Many examples and exercises

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  - Semantic tableau: global, goal-oriented, “backward”

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  - Many examples and exercises
- Two styles of inference systems
  - Resolution: local, “forward”
  - Semantic tableau: global, goal-oriented, “backward”
- Important basic properties
  - Soundness $\leadsto$ no false conclusions are drawn
  - Completeness $\leadsto$ all true conclusions are drawn
  - Efficiency $\leadsto$ avoid unnecessary inferences

Modern Resolution Framework

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- Modern resolution framework = an extension of basic resolution calculus with:
  - Powerful search control mechanisms
    - $\leadsto$ ordering and selection refinements
  - General notion of redundancy
    - $\leadsto$ simplification and optimisation techniques
  - optimised transformations into clausal form

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  - optimised transformations into clausal form
- Has many uses and applications
  - This course: verfication of Neuman-Stubblebine key exchange protocol
- Fast implementations: Vampire, (M)SPASS

Semantic tableau

- Given by a set of inference rules, e.g.:

$F \wedge G$

F

G

$F \vee G$

$F \qquad G$

- Used to construct derivation trees
- Basis for semantic tableau provers

Topics of Current Research

- Developing practical decision procedures
- Handling specific theories (equality, transitive relations, . . .) or logics (description logics, modal logics, . . .)
- Implementing fast automated theorem provers
- Relationship between different proof methods (resolution & tableau, . . .)
- Combining different proof methods and different provers
- Specific applications:
  - Software engineering
  - Ontologies and the semantic web
  - Multi-agent systems

Lectures:

- include Examples Classes
- paper-based Exercises (some assessed)

Labs:

- Approximately 35% of Teaching Time is lab
- Prolog
  - build a resolution theorem-prover
  - extend with advanced techniques
- try out MSPASS, Vampire

Reading List

- 'Course Text':

  Kelly, J. (1997), The Essence of Logic. Prentice Hall.

- Recommended:

  Schöning, U. (1989), Logic for Computer Scientists. Birkhäuser.

  Fitting, M. (1990), First-Order Logic and Automated Theorem Proving. Springer.

Assessment

- Examination (40%)
  - open book
- Exercises and labs (60%)