Craig-Interpolation (cont'd)

Proof of Property 17:
Transform F and $\neg G$ into CNF.
Let N and M, resp., denote the resulting clause sets.
Choose any atom ordering $\succ$ for which the prop. variables that occur in F but not in G are maximal.
Saturate N wrt. $Res_S^{\succ}$ (with empty selection function S) to get N^*. Let
$N' = N^* \setminus \{C \mid C$ contains a symbol in F but not in $G\}$.
I.e. $C \in N'$ iff $C \in N^*$ and C contains only symbols in G.
Let $H = \bigwedge N'$. Then, clearly $F \models H$. (Why?)
To see that $H \models G$, take $N^* \cup M$ and saturate wrt. $Res_S^{\succ}$.
This derives $\perp$, but no inferences are performed on clauses in $N^* \setminus N'$.
This implies $N' \cup M \models \perp$ and therefore $H \models G$.

Hyperresolution

- There are many variants of resolution.
  (Refer to Bachmair and Ganzinger (2001), "Resolution Theorem Proving", for further reading.)
- One well-known example is hyperresolution (Robinson 1965):
  - Assume that several negative literals are selected in a clause D.
    If we perform an inference with D, then one of the selected literals is eliminated.
  - Suppose that the remaining selected literals of D are again selected in the conclusion.
  - Then we will eliminate the remaining selected literals one by one by further resolution steps.

Hyperresolution (cont'd)

- Hyperresolution replaces these successive steps by a single inference.
- As for $Res_S^{\succ}$, the calculus is parameterised by an atom ordering $\succ$ and a selection function S.
- But S is the 'maximal' selection function, i.e. selects all negative literals in a clause.

Hyperresolution (cont'd)

- **Hyperresolution calculus** *HRes*

$$\frac{C_1 \vee A_1 \quad \ldots \quad C_n \vee A_n \qquad \neg B_1 \vee \ldots \vee \neg B_n \vee D}{(C_1 \vee \ldots \vee C_n \vee D)\sigma}$$

  provided σ is the mgu s.t. $A_1\sigma = B_1\sigma, \ldots, A_n\sigma = B_n\sigma$, and
  (i) $A_i\sigma$ strictly maximal in $C_i\sigma$, $1 \leq i \leq n$;
  (ii) nothing is selected in C_i (i.e. C_i is positive);
  (iii) the indicated $\neg B_i$ are exactly the ones selected by S, and D is positive.

- Similarly as for resolution, hyperresolution has to be complemented by a factoring rule. I.e. the ordered positive factoring rule from before.