Summary

- ground and non-ground maximality of literal wrt. clause
- ordered resolution with selection $Res_S^{\succ}$
  - inferences limited by ordering $\succ$
  - inferences limited by selection function S
- soundness and refutational completeness

COMP60121: Automated Reasoning II

Lecture 7

Previously . . .

- Ways of limiting inferences & enhancing efficiency
  - ordering restriction $\succ$
  - selection function S
- Idea:
  - Inferences restricted to $\succ$-maximal or S-selected literals
  - S overrides $\succ$
- ground and non-ground maximality of literal wrt. clause

Application: Compactness of Propositional Logic

- A theoretical application of the refutational completeness of Res is compactness of propositional logic.
- Observe: If $N \vdash_{Cal} F$ then
  there exist $F_1, \ldots, F_n \in N$ s.t. $F_1, \ldots, F_n \vdash_{Cal} F$
  (resembles compactness).
  If $N \vdash_{Res} \bot$ then
  there exist $C_1, \ldots, C_n \in N$ s.t. $C_1, \ldots, C_n \vdash_{Res} \bot$

  Note, N can be an infinite set of formulae.
- Recall

$$N \text{ is unsatisfiable} \quad \text{iff} \quad \text{for any interpretation } I,\ I \not\models N.$$

$$I \not\models N \quad \text{iff} \quad \text{for some } C \in N,\ I \not\models C$$

Compactness of Propositional Logic

Property 16 (Compactness)

Let N be a set of propositional clauses. Then:

N is unsatisfiable iff there exists a finite subset $M \subseteq N$ which is unsatisfiable.

Proof:

"$\Leftarrow$": trivial (why?).

"$\Rightarrow$": Let N be unsatisfiable, i.e. $N \models \bot$

$\Rightarrow$ $Res^*(N)$ unsatisfiable, since $N \subseteq Res^*(N)$

$\Rightarrow$ $\bot \in Res^*(N)$ by refutational completeness of resolution

$\Rightarrow$ $\exists n \geq 0$ s.t. $\bot \in Res^n(N)$

$\Rightarrow$ $\bot$ has a finite resolution proof Π;
let M be the (finite) set of assumptions in Π.

Application: Craig-Interpolation

- Another application of ordered resolution is Craig-Interpolation, which has practically relevance for distributed reasoning.

Property 17 (Craig 1957)

Let F and G be two propositional formulae such that $F \models G$. Then there exists a formula H, such that

(i) $F \models H$ and $H \models G$, and

(ii) each propositional symbol occurring in H occurs in both F and G.

- H is called an interpolant for $F \models G$
- The theorem also holds for first-order formulae. In the general case, a proof based on resolution technology is more complicated because of Skolemisation.

Craig-Interpolation (cont'd)

Proof of Property 17:

Transform F and $\neg G$ into CNF, without using structural transf.

Let N and M, resp., denote the resulting clause sets.

Choose any atom ordering $\succ$ for which the prop. variables that occur in F but not in G are maximal.

Saturate N wrt. $Res_S^{\succ}$ (with empty selection function S) to get N^*. Let

$N' = N^* \setminus \{C \mid C$ contains a symbol in F but not in $G\}$.

I.e. $C \in N'$ iff $C \in N^*$ and C contains only symbols in G.

Let $H = \bigwedge N'$. Then, clearly $F \models H$. (Why?)

To see that $H \models G$, take $N^* \cup M$ and saturate wrt. $Res_S^{\succ}$.

This derives $\bot$, but no inferences are performed on clauses in $N^* \setminus N'$.

This implies $N' \cup M \models \bot$ and therefore $H \models G$.

Hyperresolution

- There are many variants of resolution.
  (Refer to Bachmair and Ganzinger (2001), "Resolution Theorem Proving", for further reading.)
- One well-known example is hyperresolution (Robinson 1965):
  - Assume that several negative literals are selected in a clause D.
    If we perform an inference with D, then one of the selected literals is eliminated.
  - Suppose that the remaining selected literals of D are again selected in the conclusion.
  - Then we will eliminate the remaining selected literals one by one by further resolution steps.

Hyperresolution (cont'd)

- Hyperresolution replaces these successive steps by a single inference.
- As for $Res^{\succ}_{S}$, the calculus is parameterised by an atom ordering $\succ$ and a selection function S.
- But S is the 'maximal' selection function, i.e. selects all negative literals in a clause.

Hyperresolution (cont'd)

- **Hyperresolution calculus** *HRes*

$$\frac{C_1 \vee A_1 \quad \ldots \quad C_n \vee A_n \qquad \neg B_1 \vee \ldots \vee \neg B_n \vee D}{(C_1 \vee \ldots \vee C_n \vee D)\boldsymbol{\sigma}}$$

provided $\boldsymbol{\sigma}$ is the mgu s.t. $A_1\boldsymbol{\sigma} = B_1\boldsymbol{\sigma}, \ldots, A_n\boldsymbol{\sigma} = B_n\boldsymbol{\sigma}$, and

(i) $A_i\boldsymbol{\sigma}$ strictly maximal in $C_i\boldsymbol{\sigma}$, $1 \leq i \leq n$;

(ii) nothing is selected in C_i (i.e. C_i is positive);

(iii) the indicated $\neg B_i$ are exactly the ones selected by S, and D is positive.

- Similarly as for resolution, hyperresolution has to be complemented by a factoring rule. I.e. the ordered positive factoring rule from before.

Hyperresolution (cont'd)

- Note
  1. The positive clauses need not all be different.
  2. We regard clauses as equal if they are equal modulo variable renaming.
  3. $\boldsymbol{\sigma}$ is a simultaneous unifier.
- As we have seen, hyperresolution can be simulated by iterated binary resolution.
- However this yields intermediate clauses which *HRes* might not derive, and many of them might not be extendable into a full *HRes* inference.

Exercise

- Let $P \succ R \succ Q$ and consider:

$$P \vee Q$$
$$R \vee Q$$
$$\neg R \vee \neg P$$

- Give a *HRes* derivation.
- What does the derivation look like under a different ordering? $P \succ Q \succ R$?

Exercise

- Let $P \succ R \succ Q$ and consider:
$$P \vee Q$$
$$R \vee Q$$
$$\neg R \vee \neg P$$
- Give a *HRes* derivation.

1. $\underline{P} \vee Q$	given	4. $\underline{Q} \vee \underline{Q}$ (1, 2, 3, *HRes*)
2. $\underline{R} \vee Q$	given	5. $\underline{Q}$ (4, *Fact*)
3. $\boxed{\neg R} \vee \boxed{\underline{\neg P}}$	given	

- What does the derivation look like under a different ordering? $P \succ Q \succ R$?

Summary

- Applications:
  - Compactness of propositional logic
  - Craig Interpolation of propositional logic
- Hyperresolution *HRes*

  = ordered resolution with maximal selection

Exercise

- Let $P \succ R \succ Q$ and consider:
$$P \vee Q$$
$$R \vee Q$$
$$\neg R \vee \neg P$$
- Give a *HRes* derivation.

1. $\underline{P} \vee Q$	given	4. $\underline{Q} \vee \underline{Q}$ (1, 2, 3, *HRes*)
2. $\underline{R} \vee Q$	given	5. $\underline{Q}$ (4, *Fact*)
3. $\boxed{\neg R} \vee \boxed{\underline{\neg P}}$	given	

- What does the derivation look like under a different ordering? $P \succ Q \succ R$?

1. $\underline{P} \vee Q$	given
2. $R \vee \underline{Q}$	given
3. $\boxed{\neg R} \vee \boxed{\underline{\neg P}}$	given

no inference step possible

COMP60121: Automated Reasoning II

Lecture 8

Previously ...

- Compactness and Craig interpolation of propositional logic
- Ordered resolution with selection
- One variation:
  - Hyperresolution *HRes*

    = ordered resolution with maximal selection

Redundancy

- Ordering and selection functions provide local restrictions of the resolution inference rules. They limit inferences to certain literals in clauses.
- What about not performing inferences with clauses altogether? Is it also possible to just delete clauses? Under which circumstances are clauses unnecessary? (Conjecture: e. g., if they are tautologies or if they are subsumed by other clauses.)
- Intuition: Inferences with redundant clauses are not needed.

A Formal Notion of Redundancy

- Let N be a set of ground clauses and C a ground clause (not necessarily in N). C is called redundant wrt. N, if there exist $C_1, \dots, C_n \in N$, $n \geq 0$, such that
  (i) all $C_i \prec C$, and
  (ii) $C_1, \dots, C_n \models C$.
- Redundancy for general clauses:
  C is called redundant wrt. N, if
  all ground instances $C\sigma$ of C are redundant wrt. $G_\Sigma(N)$.
- C is redundant in N iff C is redundant wrt. $N \setminus \{C\}$.
- Intuition: Redundant clauses are neither minimal exceptions nor productive. Note, the converse is not always true.
- Note: The same ordering $\succ$ is used for ordering restrictions and for redundancy (and for the completeness proof).

Examples of Redundancy

Property 18

(i) C tautology (i.e., $\models C$) $\Rightarrow$ C redundant wrt. any set N.

(ii) $C\sigma \subset D$ $\Rightarrow$ D redundant wrt. $N \cup \{C\}$

(iii) $C\sigma \subseteq D$ $\Rightarrow$ $D \vee \overline{L}\sigma$ redundant wrt. $N \cup \{C \vee L,\ D\}$, where $\overline{L}$ denotes the complement of L

- When $C\sigma \subset D$ for some σ we say that D is strictly subsumed by C. (Under certain conditions one may also use non-strict subsumption, but this requires a slightly more complicated definition of redundancy.)
- $D \vee \overline{L}\sigma$ is redundant wrt. any set containing D.

Saturation up to Redundancy

- Let $Red(N)$ denote the set of clauses redundant wrt. N.
- Recall, N is saturated wrt. $Res_S^{\succ}$ iff $Res_S^{\succ}(N) \subseteq N$.
- N is called saturated up to redundancy. (wrt. $Res_S^{\succ}$) iff

$$Res_S^{\succ}(N \setminus Red(N)) \subseteq N \cup Red(N)$$

In words: every conclusion of an $Res_S^{\succ}$-inference with non-redundant clauses in N is in N or is redundant.

Property 19 ($Res_S^{\succ}$ mod. redundancy is sound & ref. complete)
Let N be saturated up to redundancy (wrt. $Res_S^{\succ}$). Then

$$N \models \bot \text{ iff } \bot \in N.$$

Saturation up to Redundancy (cont'd)

Proof (Sketch):
Ground case:

- consider construction of candidate model $I_N^{\succ}$ for $Res_S^{\succ}$
- redundant clauses are not productive
- redundant clauses in N are not minimal exceptions for $I_N^{\succ}$

The premises of "essential" inferences are either minimal exceptions or productive.

Preservation/Monotonicity Properties of Redundancy

Property 20
(i) $N \subseteq M \;\Rightarrow\; Red(N) \subseteq Red(M)$
(ii) $M \subseteq Red(N) \;\Rightarrow\; Red(N) \subseteq Red(N \setminus M)$

Proof: Exercise.

- This says that redundancy is preserved when, during a theorem proving process,
  (i) one adds (derives) new clauses or
  (ii) one deletes redundant clauses.

A Resolution Prover

- So far: static view on completeness of resolution:
  - Saturated sets are unsatisfiable iff they contain $\bot$.
- We will now consider a dynamic view:
  - How can we get saturated sets in practice?
  - The Properties 19 and 20 are the basis for the completeness proof of our prover *RP*.

Rules for Simplifications and Deletion

- We want to employ the following rules for simplification of prover states N:
  - Deletion of tautologies

$$N \cup \{C \vee A \vee \neg A\} \ \rhd \ N$$

  - Deletion of subsumed clauses

$$N \cup \{C, D\} \ \rhd \ N \cup \{C\}$$

  if $C\sigma \subseteq D$ (C subsumes D).

  - Reduction (also called subsumption resolution)

$$N \cup \{D \vee L,\ C \vee D\sigma \vee \overline{L}\sigma\} \ \rhd \ N \cup \{D \vee L,\ C \vee D\sigma\}$$

$$N \cup \{C \vee L,\ D \vee C\sigma \vee \overline{L}\sigma\} \ \rhd \ N \cup \{C \vee L,\ D \vee C\sigma\}$$

Resolution Prover *RP*

- **3 clause sets:**
  - **N**(ew) containing new conclusions
  - **P**(rocessed) containing simplified resolvents
  - **O**(ld) where clause are put once their inferences have been computed
- **Strategy:** Inferences will only be computed when there are no possibilities for simplification

Transition Rules for *RP* (I)

Tautology elimination

$$\boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \mid \boldsymbol{O} \quad \rhd \quad \boldsymbol{N} \mid \boldsymbol{P} \mid \boldsymbol{O}$$

if C is a tautology

Forward subsumption

$$\boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \mid \boldsymbol{O} \quad \rhd \quad \boldsymbol{N} \mid \boldsymbol{P} \mid \boldsymbol{O}$$

if some $D \in \boldsymbol{P} \cup \boldsymbol{O}$ subsumes C

Backward subsumption

$$\boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \cup \{D\} \mid \boldsymbol{O} \quad \rhd \quad \boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \mid \boldsymbol{O}$$

$$\boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \mid \boldsymbol{O} \cup \{D\} \quad \rhd \quad \boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \mid \boldsymbol{O}$$

if C strictly subsumes D

Transition Rules for *RP* (II)

Forward reduction

$$\boldsymbol{N} \cup \{D \vee L\} \mid \boldsymbol{P} \mid \boldsymbol{O} \quad \rhd \quad \boldsymbol{N} \cup \{D\} \mid \boldsymbol{P} \mid \boldsymbol{O}$$

if there exists $C \vee L' \in \boldsymbol{P} \cup \boldsymbol{O}$

such that $\overline{L} = L'\sigma$ and $C\sigma \subseteq D$

Backward reduction

$$\boldsymbol{N} \mid \boldsymbol{P} \cup \{C \vee L\} \mid \boldsymbol{O} \quad \rhd \quad \boldsymbol{N} \mid \boldsymbol{P} \cup \{C\} \mid \boldsymbol{O}$$

$$\boldsymbol{N} \mid \boldsymbol{P} \mid \boldsymbol{O} \cup \{C \vee L\} \quad \rhd \quad \boldsymbol{N} \mid \boldsymbol{P} \mid \boldsymbol{O} \cup \{C\}$$

if there exists $D \vee L' \in \boldsymbol{N}$

such that $\overline{L} = L'\sigma$ and $D\sigma \subseteq C$

Transition Rules for *RP* (III)

Clause processing

$$\boldsymbol{N} \cup \{C\} \mid \boldsymbol{P} \mid \boldsymbol{O} \quad \triangleright \quad \boldsymbol{N} \mid \boldsymbol{P} \cup \{C\} \mid \boldsymbol{O}$$

Inference computation

$$\emptyset \mid \boldsymbol{P} \cup \{C\} \mid \boldsymbol{O} \quad \triangleright \quad \boldsymbol{N} \mid \boldsymbol{P} \mid \boldsymbol{O} \cup \{C\}, \quad \text{with } \boldsymbol{N} = \mathit{Res}_S^{\succ}(\boldsymbol{O} \cup \{C\})$$

Soundness and Completeness

Property 21

$$N \models \bot \quad \text{iff} \quad N \mid \emptyset \mid \emptyset \overset{*}{\triangleright} N' \cup \{\bot\} \mid \_ \mid \_$$

Proof in
L. Bachmair, H. Ganzinger (2001), "Resolution Theorem Proving"
(on H. Ganzinger's Web page under Publications/Journals;
published in "Handbook on Automated Reasoning").

Fairness

- Problem:
  - If N is unsatisfiable, then $N \mid \emptyset \mid \emptyset \overset{*}{\triangleright} N' \cup \{\bot\} \mid \_ \mid \_$.
    This states the existence of a finite derivation of $\bot$.
  - Does this imply that every derivation starting from an unsatisfiable set N eventually produces $\bot$?
  - No: a clause could be kept in $\boldsymbol{P}$ without ever being used for an inference.

Fairness (cont'd)

- We need in addition a fairness condition:
  - If an inference is possible forever (that is, none of its premises is ever deleted), then it must be performed eventually.
- One possible way to guarantee fairness:
  Implement $\boldsymbol{P}$ as a queue
  (there are other techniques to guarantee fairness).
- With this additional requirement, we get a stronger result:
  If N is unsatisfiable, then every fair derivation will eventually produce $\bot$.

Summary

- General notion redundancy
  - Justifies deletion of clauses
  - Standard instances:
    tautology deletion, strict subsumption deletion
- Saturation up to redundancy
- Soundness & completeness of $Res_S^{\succ}$ modulo redundancy
- Implementing a (sound & complete) resolution prover
  - Specific redundancy elimination & simplification rules
  - Transition rules for deduction, deletion & simplification steps
  - Fairness

COMP60121: Automated Reasoning II

Lecture 9

Previously . . .

- Advanced techniques of resolution theorem proving
  - optimised transformations to clause form
  - ordering restriction $\succ$
  - selection function S
  - redundancy elimination
- Powerful and versatile framework
- Implementing a resolution theorem prover

Example: Neuman-Stubblebine Protocol

- Formalisation of a concrete application:
  Neuman-Stubblebine key exchange protocol.
- Proof by refutation:
  unsatisfiable $\Rightarrow$ intruder can break the protocol.
- Proof by consistency:
  consistency $\Rightarrow$ no unsafe states exist.
- Non-termination $\Rightarrow$ potential attack on the protocol.
- Termination requires elimination of redundancy.

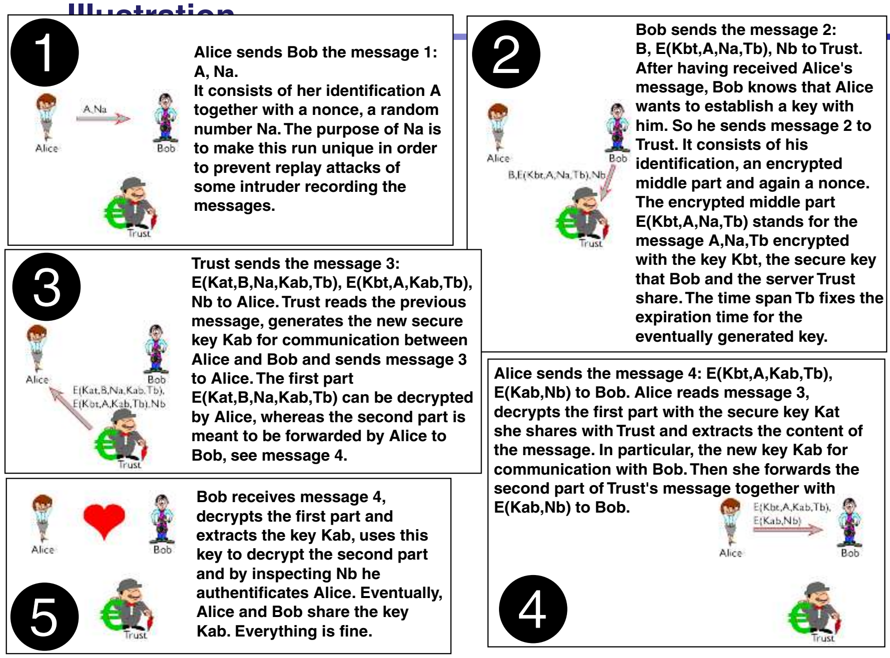
The Problem

- Automatic Analysis of Security Protocols using SPASS: An Automated Theorem Prover for First-Order Logic with Equality, by Christoph Weidenbach
- The growing importance of the internet causes a growing need for security protocols that protect transactions and communication. It turns out that the design of such protocols is highly error-prone. Therefore, there is a need for tools that automatically detect flaws like, e.g., attacks by an intruder.
- Here we show that our automated theorem prover SPASS can be used successfully to analyze the Neuman-Stubblebine key exchange protocol [1]. To this end the protocol is formalized in logic and then the security properties are automatically analyzed by SPASS. A detailed description of the analysis can be found in [2].

The Problem (cont'd)

- The animation successively shows two runs of the Neuman-Stubblebine [1] key exchange protocol. The first run works the way the protocol is designed to do, i.e. it establishes a secure key K_{ab} between Alice and Bob.
- The second run shows a potential problem of the protocol. An intruder may intercept the final message sent from Alice to Bob, replace it with a different message and may eventually own a key N_a that Bob believes to be a secure key with Alice.
- The initial situation for the protocol is that the two participants Alice and Bob want to establish a secure key for communication among them. They do so with the help of a trusted key server, Trust, where both already have a secure key for communication with Trust.
- The next picture shows a sequence of four message exchanges that eventually establishes the key.

Illustration



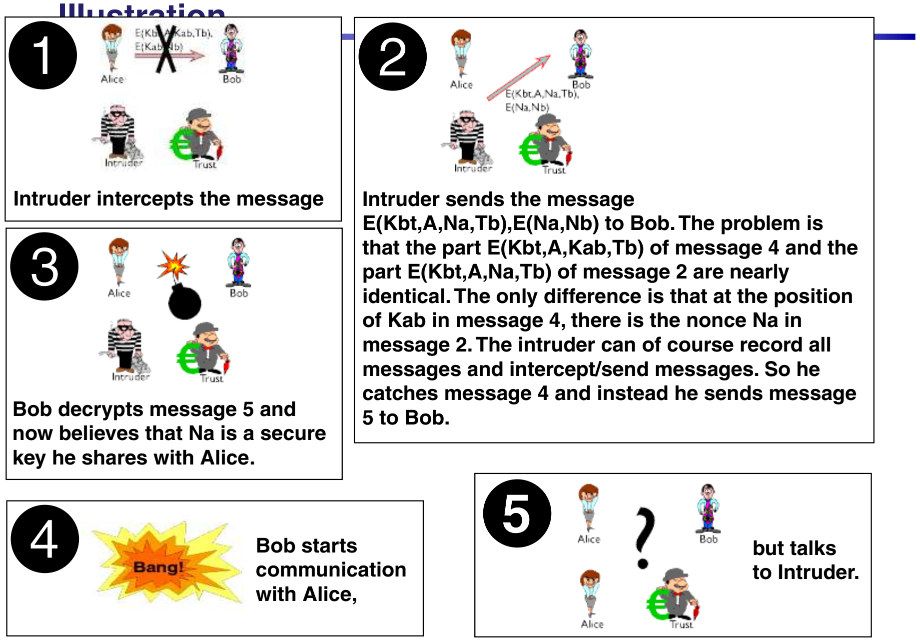
Neuman-Stubblebine: A Regular Run

1. $A \xrightarrow{A,\, N_a} B$
2. $B \xrightarrow{B,\, N_b,\, E_{K_{bt}}(A,\, N_a,\, T_b)} T$
3. $T \xrightarrow{E_{K_{at}}(B,\, N_a,\, K_{ab},\, T_b),\, E_{K_{bt}}(A,\, K_{ab},\, T_b),\, N_b} A$
4. $A \xrightarrow{E_{K_{bt}}(A,\, K_{ab},\, T_b),\, E_{K_{ab}}(N_b)} B$

What Can Happen?

- How can an intruder now break this protocol?
  - The key K_{ab} is only transmitted inside encrypted parts of messages and
  - we assume that an intruder cannot break any keys nor does it know any of the initial keys K_{at} or K_{bt}.
- Here is how ...

Illustration



Breaking the Neuman-Stubblebine Protocol

1. $A \xrightarrow{A,\, N_a} B$
2. $B \xrightarrow{B,\, N_b,\, E_{K_{bt}}(A,\, N_a,\, T_b)} T$
3. $T \xrightarrow{E_{K_{at}}(B,\, N_a,\, K_{ab},\, T_b),\, E_{K_{bt}}(A,\, K_{ab},\, T_b),\, N_b} A$
4. $A \xrightarrow{E_{K_{bt}}(A,\, K_{ab},\, T_b),\, E_{K_{ab}}(N_b)} B$
3. $I \xrightarrow{E_{K_{bt}}(A,\, N_a,\, T_b),\, E_{N_a}(N_b)} B$

The Formalisation

- The key idea of the formalisation is to describe the set of sent messages.
- This is done by introducing a monadic predicate M in first-order logic.
- Furthermore, every participant holds its set of known keys, represented by the predicates Ak for Alice's keys, Bk for Bob's keys, Tk for Trust's keys and Ik for the keys the intruder knows.
  (The remaining symbols are introduced and explained with their first appearance in a formula.)
- Then the four messages can be translated into the following formulae ...

The Formalisation: Step 1

- **Step 1)** $A \longrightarrow B : A, Na$

$$Ak(key(at, t)) \tag{1}$$

$$M(sent(a, b, pair(a, na))) \tag{2}$$

- (1) expresses that initially Alice holds the *key* *at* for communication with *t* (for Trust). (2) states that she sends the first message.
- In order to formalize messages we employ a three place function sent where the first argument is the sender, the second the receiver and the third the content of the message.
- So the constant *a* represents Alice, *b* Bob, *t* Trust and *i* Intruder.
- The functions *pair* (*triple*, *quadr*) simply form sequences of messages of the indicated length.

The Formalisation: Step 2

- **Step 1)** $A \longrightarrow B : A, Na$
  Step 2) $B \longrightarrow T : B, Nb, E_{Kbt}(A, Na, Tb)$

$$Bk(key(bt, t)) \tag{3}$$

$$\forall xa, xna\ [M(sent(xa, \underline{b}, pair(xa, xna))) \longrightarrow M(sent(b, t, triple(b, \underline{nb(xna)}, encr(triple(xa, xna, \underline{tb(xna)}), bt)))))] \tag{4}$$

- Bob holds the key *bt* for secure communication with Trust and whenever he receives a message of the form of message 1 (formula (2)), he sends a key request to Trust according to message 2.
- Note, encryption is formalized by the two place function *encr* where the first argument is the message and the second argument the key.
- Every lowercase symbol starting with an *x* denotes a variable. The functions *nb* and *tb* generate, respectively, a new nonce and time span out of *xa*'s (Alice's) request represented by her nonce *xna*.

The Formalisation: Step 3

- **Step 2)** $B \longrightarrow T : B, Nb, E_{Kbt}(A, Na, Tb)$
  Step 3) $T \longrightarrow A : E_{Kat}(B, Na, Kab, Tb),\ E_{Kbt}(A, Kab, Tb),\ Nb$

$$Tk(key(at, a))) \wedge Tk(key(bt, b)) \tag{5}$$

$$\begin{aligned} &\forall xb, xnb, xa, xna, xbet, xbt, xat, xk \\ &[\ (M(sent(xb, \underline{t}, triple(xb, xnb, encr(triple(xa, xna, xbet), xbt)))) \\ &\quad \wedge \underline{Tk(key(xbt, xb))} \wedge \underline{Tk(key(xat, xa))}) \\ &\quad \longrightarrow M(sent(t, xa, triple(encr(quadr(xb, xna, \underline{kt(xna)}, xbet), xat), \\ &\qquad encr(triple(xa, kt(xna), xbet), xbt), xnb)))\ ] \end{aligned} \tag{6}$$

- Trust holds the keys for Alice and Bob and answers appropriately to a message in the format of message 2.
- Note: decryption is formalized by unification with an appropriate term where it is checked that the necessary keys are known to Trust.
- The server generates the key by applying his key generation function *kt* to the nonce *xna*.

The Formalisation: Step 4

- **Step 3)** $T \longrightarrow A : E_{Kat}(B, Na, Kab, Tb),\ E_{Kbt}(A, Kab, Tb),\ Nb$
  Step 4) $A \longrightarrow B : E_{Kbt}(A, Kab, Tb),\ E_{Kab}(Nb)$

$$\begin{aligned} &\forall xnb, xbet, xk, xm, xb, xna \\ &[\ M(sent(\underline{t}, \underline{a}, triple(encr(quadr(xb, xna, xk, xbet), at), xm, xnb)) \\ &\quad \longrightarrow (M(sent(a, xb, pair(xm, encr(xnb, xk)))) \wedge Ak(key(xk, xb)))\ ] \end{aligned} \tag{7}$$

$$\begin{aligned} &\forall xbet, xk, xnb, xa, xna \\ &[\ M(sent(xa, \underline{b}, pair(encr(triple(xa, xk, \underline{tb(xna)}), bt), \\ &\qquad encr(\underline{nb(xna)}, xk))) \longrightarrow Bk(key(xk, xa))] \end{aligned} \tag{8}$$

- Finally, Alice answers according to the protocol to message 3 and stores the generated key for communication, formula (7).
- Formula (8) describes Bob's behaviour when he receives Alice's message. Bob decodes this message and stores the new key as well.

A's Formalisation Part I

$$A \xrightarrow{A,\ N_a} B$$

$$P(a)$$
$$Ak(key(at, t))$$
$$M(sent(a, b, pair(a, na)))$$
$$Sa(pair(b, na))$$

- Sa is Alice's local store that will eventually be used to verify the nonce when it is sent back to her in Step (3).

A's Formalisation Part II

$$T \xrightarrow{E_{K_{at}}(B, N_a, K_{ab}, T_b),\ E_{K_{bt}}(A, K_{ab}, T_b),\ N_b} A$$
$$A \xrightarrow{E_{K_{bt}}(A, K_{ab}, T_b),\ E_{K_{ab}}(N_b)} B$$

$$\forall xb, xna, xnb, xk, xbet, xm$$
$$[\, M(sent(\underline{t}, \underline{a}, triple(encr(quadr(xb, xna, xk, xbet), at), xm, xnb)))$$
$$\wedge\ \underline{Sa(pair(xb, xna))}$$
$$\longrightarrow$$
$$M(sent(a, xb, pair(xm, encr(xnb, xk))))$$
$$\wedge\ Ak(key(xk, xb))\,]$$

The Intruder

- The Intruder is modeled as an exhaustive, active attacker. It records all messages, decomposes the messages as far as possible and generates all possible new compositions.
- Furthermore, any object it has in hand is considered as a key and tried for encryption as well as for decryption.
- All these messages are posted. The set of messages the intruder has available is represented by the predicate *Im*.
- The participants are Alice, Bob, Trust and Intruder:

$$P(a) \wedge P(b) \wedge P(t) \wedge P(i) \tag{9}$$

- The intruder records all messages:

$$\forall xa, xb, xm\ [M(sent(xa, xb, xm)) \longrightarrow Im(xm)] \tag{10}$$

The Intruder

- It decomposes and decrypts all messages it owns the key for:

$$\forall u, v\ [Im(pair(u, v)) \longrightarrow Im(u) \wedge Im(v)] \tag{11}$$
$$\forall u, v, w\ [Im(triple(u, v, w)) \longrightarrow Im(u) \wedge Im(v) \wedge Im(w)] \tag{12}$$
$$\forall u, v, w, z\ [Im(quadr(u, v, w, z)) \longrightarrow Im(u) \wedge Im(v) \wedge Im(w) \wedge Im(z)] \tag{13}$$
$$\forall u, v, w\ [Im(encr(u, v)) \wedge Ik(key(v, w)) \longrightarrow Im(u)] \tag{14}$$

- It composes all possible messages:

$$\forall u, v\ [Im(u) \wedge Im(v) \longrightarrow Im(pair(u, v))] \tag{15}$$
$$\forall u, v, w\ [Im(u) \wedge Im(v) \wedge Im(w) \longrightarrow Im(triple(u, v, w))] \tag{16}$$
$$\forall u, v, w, x\ [Im(u) \wedge Im(v) \wedge Im(w) \wedge Im(x) \longrightarrow Im(quadr(u, v, w, x))] \tag{17}$$

The Intruder

- It considers every item to be a key and uses it for encryption:

$$\forall v, w\ [Im(v) \wedge P(w) \longrightarrow Ik(key(v, w))] \quad (18)$$

$$\forall u, v, w\ [Im(u) \wedge Ik(key(v, w)) \wedge P(w) \longrightarrow Im(encr(u, v))] \quad (19)$$

- It sends everything:

$$\forall x, y, u\ [P(x) \wedge P(y) \wedge Im(u) \longrightarrow M(sent(x, y, u))] \quad (20)$$

- Finally we must formalize the insecurity requirement. Intruder must not have any key for communication with Bob that Bob believes to be a secure key for Alice:

$$\exists x\ [Ik(key(x, b)) \wedge Bk(key(x, a))]$$

This actually states the opposite, i.e. that an attack exists. Thus finding a proof means there is a vulnerability.

SPASS Solves the Problem

- Now the protocol formulae together with the intruder formulae (9)-(20) and the insecurity formula above can be given to SPASS. Then SPASS automatically proves that this formula holds and that the problematic key is the nonce *Na*.
- The protocol can be repaired by putting type checks on the keys, such that keys can no longer be confused with nonces. This can be added to the SPASS first-order logic formalisation.
  Then SPASS disproves the insecurity formula above.
- The type checking capability is currently unique to SPASS. Although some other provers might be able to prove that the insecurity formula holds in the formalisation without type checks, we are currently not aware of any prover that can disprove the insecurity formula in the formalisation with type checking.
- Further details can be found in [2], below. The experiment is available in full detail from the SPASS home page in the download area or the COMP6012 directory.

References

[1] Neuman, B. C. and Stubblebine, S. G., 1993, A note on the use of timestamps as nonces, ACM SIGOPS, Operating Systems Review, 27(2), 10-14.

[2] Weidenbach, C., 1999, Towards an automatic analysis of security protocols in first-order logic, in 16th International Conference on Automated Deduction, CADE-16, Vol. 1632 of LNAI, Springer, pp. 378-382.

Summary: Resolution Theorem Proving

- Resolution is a machine calculus.
- Parameters:
  - atom ordering $\succ$ and selection function S.
  - On the non-ground level, ordering constraints can (only) be solved approximatively.
- Completeness proof by constructing candidate models
  - from productive clauses $C \vee A$, $A \succ C$; inferences with these reduce exceptions.

Summary: Resolution Theorem Proving (cont'd)

- Local restrictions of inferences via $\succ$ and S
  - $\Rightarrow$ fewer proof variants;
  - $\Rightarrow$ justification for numerous standard refinements;
  - $\Rightarrow$ simulation of certain kinds of tableaux and other proof methods.
- Global restrictions of the search space via elimination of redundancy
  - $\Rightarrow$ computing with "smaller" clause sets;
  - $\Rightarrow$ termination on many expressive, decidable fragments.
- Further specialisation of inference systems required for reasoning with orderings, equality and specific algebraic theories (lattices, abelian groups, rings, fields), integers

Current research

- AR for specific theories: $\approx$, transitive relations, arithmetic, ...
- Resolution decision procedures for solvable fragments of f.-o. logic
- Implementing fast automated theorem provers (optimisations, heuristics, experimentation, ...)
- Making AR tools more intelligent
- Synthesis and analysis of different proof methods
- Platforms for combinations of reasoning tools: other provers (e.g. h.o. provers, theory decision procedures), model checkers, ..., SMT
- AR with distributed, heterogeneous, dynamic information
- AR for description logics, ontologies and the semantic web
- AR for modal logics, multi-agent systems
- Security protocol analysis; soft-/hardware design & verification
- ...

Research at Manchester (incomplete summary)

- automated theorem proving
  - resolution
  - tableau
  - other methods
- decision procedures
- model generation, s.o. quantifier elimination
- verification, model checking
- first-order logic, modal logics, description logics, program logics, temporal logics
- Vampire, SPASS, SCAN, ... FACT++, ICOM, KAON2, ...
- reasoning for semantic web, ontologies
- AR for natural language processing

COMP60121: Automated Reasoning II

Appendix

Basic Notions

- Let $\star$ be an operator defined over (elements of) a set X.
- $\star$ is commutative iff
  for any $x, y \in X$, $x \star y = y \star x$.
- $\star$ is associative iff
  for any $x, y, z \in X$, $((x \star y) \star z) = (x \star (y \star z))$.

Notation

- p, q, P, Q predicate symbols
- x, y, z variables
- a, b, c constants
- f, g function symbols
- s, t terms
- A, B atoms
- L literals
- C, D clauses
- N sets of clauses
- F, G formulae
- $F[G]$ G is subformula of F
- σ substitutions
- Σ given signature
- X given set of variables
- T_Σ terms over signature
- $x \mapsto s$ substitution of term s into variable x
- $\mathcal{M}$ f.o. interpretation (mapping)
- $\overline{L}$ complement of literal L