Orderings. Let $(X, \succ)$ be an ordering. The *multi-set extension ordering* $\succ_{\text{mul}}$ on (finite) multi-sets over X is defined by

$$\begin{aligned} S_1 \succ_{\text{mul}} S_2 \text{ iff } & S_1 \neq S_2 \text{ and} \\ & \forall x \in X, \text{ if } S_2(x) > S_1(x) \text{ then} \\ & \quad \exists y \in X : \ y \succ x \text{ and } S_1(y) > S_2(y) \end{aligned}$$

Suppose $\succ$ is a total and well-founded ordering on ground atoms. $\succ_L$ denotes the *ordering on ground literals* and is defined by:

$$\begin{array}{lll} [\neg]A & \succ_L & [\neg]B, \quad \text{if } A \succ B \\ \neg A & \succ_L & A \end{array}$$

$\succ_C$ denotes the *ordering on ground clauses* and is defined by the multi-set extension of $\succ_L$, i.e. $\succ_C = (\succ_L)_{\text{mul}}$.

Maximal literals. Let $\succ$ be a total and well-founded ordering on ground atoms.

A ground literal L is called *[strictly] maximal* wrt. a ground clause C iff for all L' in C: $L \succeq L'$ [$L \succ L'$].

A non-ground literal L is *[strictly] maximal* wrt. a (ground or non-ground) clause C iff there exists a ground substitution σ such that for all L' in C: $L\sigma \succeq L'\sigma$ [$L\sigma \succ L'\sigma$].

Herbrand models. The *Herbrand universe* (over Σ), denoted T_Σ, is the set of all ground terms over Σ.

A *Herbrand interpretation* (over Σ), denoted I, is a set of ground atoms over Σ.

Truth in I of *ground formulae* is defined inductively by:

$$\begin{array}{rll} I \models \top & & I \not\models \bot \\ I \models A & \text{iff} & A \in I, \text{ for any ground atom } A \\ I \models \neg F & \text{iff} & I \not\models F \\ I \models F \wedge G & \text{iff} & I \models F \text{ and } I \models G \\ I \models F \vee G & \text{iff} & I \models F \text{ or } I \models G \end{array}$$

Truth in I of any *quantifier-free formula* F with free variables $x_1, \ldots, x_n$ is defined by:

$$I \models F(x_1, \ldots, x_n) \ \text{ iff } \ I \models F(t_1, \ldots, t_n), \text{ for every } t_i \in \mathrm{T}_\Sigma$$

Truth in I of any *set N of clauses* is defined by:

$$I \models N \quad \text{iff} \quad I \models C, \text{ for each } C \in N$$

Construction of candidate models. Let $N, \succ$ be given.

For all ground clauses C over the given signature, the sets I_C and Δ_C are inductively defined with respect to the clause ordering $\succ$ by:

$$I_C := \bigcup_{C \succ D} \Delta_D$$

$$\Delta_C := \begin{cases} \{A\}, & \text{if } C \in N,\ \ C = C' \vee A,\ \ A \succ C' \\ & \text{and } \ I_C \not\models C \\ \emptyset, & \text{otherwise} \end{cases}$$

We say that C *produces* A, if $\Delta_C = \{A\}$.

The *candidate model* for N (wrt. $\succ$) is given as

$$I_N^{\succ} := \bigcup_{C \in N} \Delta_C.$$

We also simply write I_N, or I, for $I_N^{\succ}$, if $\succ$ is either irrelevant or known from the context.

Ordered resolution with selection calculus $Res_S^{\succ}$. Let $\succ$ be an atom ordering and S a selection function.

$$(\textit{Ordered resolution with selection rule}) \qquad \frac{C \vee A \qquad \neg B \vee D}{(C \vee D)\sigma}$$

provided $\sigma = \mathrm{mgu}(A, B)$ and

(i) $A\sigma$ strictly maximal wrt. $C\sigma$;

(ii) nothing is selected in C by S;

(iii) either $\neg B$ is selected,
or else nothing is selected in $\neg B \vee D$ and $\neg B\sigma$ is maximal wrt. $D\sigma$.

$$(\textit{Ordered factoring rule}) \qquad \frac{C \vee A \vee B}{(C \vee A)\sigma}$$

provided $\sigma = \mathrm{mgu}(A, B)$ and

(i) $A\sigma$ is maximal wrt. $C\sigma$ and

(ii) nothing is selected in C.

Hyperresolution calculus $HRes$**.**

$$(\textit{Ordered hyperresolution rule}) \quad \frac{C_1 \vee A_1 \quad \ldots \quad C_n \vee A_n \qquad \neg B_1 \vee \ldots \vee \neg B_n \vee D}{(C_1 \vee \ldots \vee C_n \vee D)\sigma}$$

provided σ is the mgu s.t. $A_1\sigma = B_1\sigma, \ldots, A_n\sigma = B_n\sigma$, and

(i) $A_i\sigma$ strictly maximal in $C_i\sigma$, $1 \le i \le n$;

(ii) nothing is selected in C_i (i.e. C_i is positive);

(iii) the indicated $\neg B_i$ are exactly the ones selected by S, and D is positive.

$$(\textit{Ordered factoring rule}) \qquad \frac{C \vee A \vee B}{(C \vee A)\sigma}$$

provided $\sigma = \text{mgu}(A, B)$ and

(i) $A\sigma$ is maximal wrt. $C\sigma$ and

(ii) nothing is selected in C.

Redundancy. Let N be a set of ground clauses and C a ground clause. C is called *redundant* wrt. N, if there exist $C_1, \ldots, C_n \in N$, $n \ge 0$, such that

(i) all $C_i \prec C$, and

(ii) $C_1, \ldots, C_n \models C$.

A general clause C is called *redundant* wrt. N, if all ground instances $C\sigma$ of C are redundant wrt. $G_\Sigma(N)$.

N is called *saturated up to redundancy* (wrt. $Res_S^{\succ}$) iff every conclusion of an $Res_S^{\succ}$-inference with non-redundant clauses in N is in N or is redundant (i.e.

$$Res_S^{\succ}(N \setminus Red(N)) \subseteq N \cup Red(N),$$

where $Red(N)$ denotes the set of clauses redundant wrt. N).