MSc Module CS616
Knowledge Representation and Reasoning
Teaching Week exercises
(Assessed Work)

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Exercise 1 (Introduction) [9 marks] Read and think about different definitions or explanations of the following three terms and present, for each of them, your favorite definition. Justify your choice.

1. knowledge — in contrast to information and data?
2. artificial intelligence
3. ontologies

Exercise 2 (FOL) [8 marks] For each of the following statements, formulate them in FOL formula:

1. every Elephant is a Mammal
2. Mammals and Birds are disjoint
3. every Student is a part-time- or a full-time-Student
4. every Student is either a part-time- or a full-time-Student, but not both
5. all bicycles have exactly two parts that are wheels, and are powered by a human
6. every human has a mother and a father
7. for any two different rational numbers, there is a third one "between" them
8. whenever a person owns a thing, then this person also owns the parts of this thing

Exercise 3 (FOL) [9 marks] Indicate, for each of the following formulae, whether it is satisfiable and whether it is valid. Explain your choice in one sentence.

1. $(\forall x.\exists y.Q(x,y)) \vee (\forall y.\exists x.\neg Q(x,y))$
2. $(\exists y.\forall x.Q(x,y)) \vee (\exists x.\forall y.Q(x,y))$
3. $(\exists x.\forall z.Q(x,z)) \wedge (\forall x.\exists y.\neg Q(x,y))$

Exercise 4 (FOL) [2 marks] Construct, for the following formulae, an equivalent one in NNF

$$\neg((\exists x.\forall y.\neg P(x,y)) \wedge (\forall z.\exists w.(R(w,z) \vee \neg R(z,w))))$$

Exercise 5 (FOL) [8 marks] Describe the rewrite rules that can be used to transform any FOL formula over connectives $\neg, \wedge, \vee$ into NNF (Hint: there should be 5 rules)

Exercise 6 (FOL) [5 marks] Use the tableau algorithm to prove the unsatisfiability of

$$(\forall x.(A(x) \wedge B(x))) \wedge \neg(\forall x.A(x))$$

Exercise 7 (FOL) [5 marks] Use the tableau algorithm to prove the validity of

$$\forall x.(P(x) \vee \neg P(x))$$

Exercise 8 (FOL) [5 marks] Use the tableau algorithm to prove that $\forall x.B(x)$ is a consequence of $\{\forall x.A(x), \forall x.A(x) \Rightarrow \forall x.B(x)\}$.

Exercise 9 (FOL) [5 marks] Explain how the tableau algorithm can be used as a "sub-program" to check whether a given formula is a consequence of a given finite set of formulae Γ.

ICOM (see http://www.inf.unibz.it/ franconi/icom/) is a tool that implements the ideas we have seen in the lecture on translating ER-diagrams into description logic TBoxes. You can download it from the above web page, or use it in the MSc labs: simply copy the file O:\icom\icom.zip into your homedirectory and unzip it to p:\icom

Exercise 10 (ICOM) [10 marks] Use ICOM to draw an ER diagramm describing students, professors, lecturers, course, study opportunities, labs, exams, etc.. Take care to add as many constraints as possible (e.g., disjoint/covering is-a links and cardinality constraints), and explain your diagram.

Exercise 11 (ICOM) [10 marks]

1. Take the ER-diagram from Exercise 10 and translate it into a set of FOL formulae. Explain how you came to the resulting formulae. You do not need to translate the diagram completely, but you should cover all aspects!

2. Add two formulae that express additional integrity constraints and explain them.

3. Add another integrity constraint that implies the unsatisfiability of one of the predicates corresponding to an entity.

Exercise 12 (ICOM) [10 marks]

1. Take the ER-diagram from the Exercise 10 and translate it into a description logic knowledge base. Explain how you came to the resulting axioms. Again, you do not need to translate the diagram completely, but you should cover all aspects!

2. Add two axioms that express additional integrity constraints and explain them (you might be able to re-use the constraints from the previous exercise).

3. Add another integrity constraint that implies the unsatisfiability of one of the predicates corresponding to an entity (you might be able to re-use the constraint from the previous exercise).

Exercise 13 (Non-Standard) [8 marks]Compute at least one concept from $\mathsf{msc}_{\mathcal{EL}}(a, \mathcal{T}, \mathcal{A})$ for

$$\begin{aligned} \mathcal{T} = \quad & \{A \doteq B \sqcap \exists r.C\} \\ \mathcal{A} = \quad & \{a : B,\ (a, b_1) : r,\ (a, b_2) : r, b_1 : C, b_2 : D\}, \end{aligned}$$

and explain your answer.

Exercise 14 (Non-Standard) [8 marks] Compute at least one concept from $\mathsf{msc}_{\mathcal{EL}}(b, \mathcal{T}, \mathcal{A})$ for

$$\begin{aligned} \mathcal{T} = \{ \quad & A \doteq B \sqcap \exists r.C \\ & C \doteq D \sqcap \exists s.E\} \\ \mathcal{A} = \{ \quad & b : B,\ c : D, (b, c) : r, (c, e) : s, e : (E \sqcap F)\}, \end{aligned}$$

explain your answer and explain why $A \sqcap \exists r.\exists s.F \notin \mathsf{msc}_{\mathcal{EL}}(b, \mathcal{T}, \mathcal{A})$

Exercise 15 (Non-Standard) Compute a concept from $\mathsf{lcs}_{\mathcal{EL}}(C, D, \emptyset)$ for

$$\begin{aligned} C = \quad & \texttt{Happy} \sqcap \texttt{Student} \sqcap \exists\texttt{r}.(\texttt{Happy} \sqcap \texttt{Student}) \sqcap \exists\texttt{r}.\texttt{Doctor} \sqcap \forall\texttt{r}.\texttt{Spoiled} \\ D = \quad & \texttt{Happy} \sqcap \exists\texttt{r}.\texttt{Spoiled} \sqcap \exists\texttt{r}.\texttt{Happy} \sqcap \forall\texttt{r}.\texttt{Doctor} \end{aligned}$$

Exercise 16 (Non-Standard) [12 marks] Construct an example TBox (with at least 4 concept definitions) and ABox (with at least 3 objects), choose 3 objects, and use the non-standard inference approach to compute a concept that covers these 3 objects. Explain your answer.

Exercise 17 (Temporal) [6 marks] Express the following statements in LTL

1. if the sun shines tomorrow, I will go swimming

2. if I get my A-levels, I will become a student and remain a student until I get a good job, and then I will become rich.

Exercise 18 (Temporal) [6 marks] Translate the following LTL formulae into English (where bf means “having breakfast”, bt “brushing teeth”, w “work”, and d “dinner”) and answer the questions.

1. $bf \wedge bf\mathcal{U}(bt \wedge \Diamond(w\mathcal{U}d))$. Does this mean that I do have to work?

2. $\Box(d \Rightarrow (\Diamond(bf \wedge w\mathcal{U}d))$. Explain why this statement does not make sense.

Exercise 19 (Temporal) [10 marks] Take the traffic light example from the lecture and extend it to describe traffic lights that have a pedestrian signal as well. Choose a mode of operation for these traffic lights and explain it. Formulate the appropriate formulae in LTL and explain them.

Exercise 20 (Temporal) [12 marks] Describe the following concepts and axioms in $\mathsf{LTL}_{\mathcal{ALC}}$:

1. Students that sleep until all their flatmates are woken up.

2. Students that will eventally work for a boss who is married to a person who will eventually become rich.

3. In all times, human beings are mortal.

4. In all times, each human being will eventually love somebody.

5. Mortal being are exactly those living beings that are eventually not alive, and then remain not alive forever.

6. Zombies are exactly those living beings that are eventually not alive, and then become alive again.

Exercise 21 (Defaults) [10 marks] Use the enhanced algorithm for the computation of extensions of propositional default theories to compute all extension of the following default theories. Explain your approach.

1.
   - $\mathcal{W} = \{\texttt{AusSwan}, \texttt{AusSwan} \Rightarrow \texttt{Swan}\}$
   - $\mathcal{D} = \{\frac{\texttt{Swan} : \texttt{White}}{\texttt{White}}, \frac{\texttt{AusSwan} : \texttt{Black}}{\texttt{Black}}\}$

2.
   - $\mathcal{W} = \{\texttt{Penguin}, \texttt{Penguin} \Rightarrow \texttt{Bird}\}$
   - $\mathcal{D} = \{\frac{\texttt{Bird} : (\texttt{Flies} \wedge \neg\texttt{Penguin})}{\texttt{Flies}}, \frac{\texttt{Penguin} : \neg\texttt{Flies}}{\neg\texttt{Flies}}\}$,

Exercise 22 (Defaults) [5 marks] You might have noticed that, in the examples, many defaults were of the form $\frac{\alpha : \beta}{\beta}$. Use the library and/or the internet to find out more about forms of defaults: what is this form called? Are there other "common" forms of defaults? Can you give an example?

Exercise 23 (Defaults) [18 marks] For each of the following six statements,

1. $a : D$ and $a : \neg D$

2. $b : D$ and $b : \neg D$

3. $c : D$ and $c : \neg D$

explain whether it is consistent with the following prioritised $\mathcal{ALC}$ default theory

$$
\begin{array}{rll}
\mathcal{T} = \{ & C_1 = & C \sqcap D \sqcap (\exists r.A) \sqcap (\forall r.B), \\
 & C_2 = & C \sqcap (\exists r.B) \sqcap (\forall r.A), \\
 & C_3 = & C \sqcap \exists r.(A \sqcap B)\} \\
\mathcal{A} = \{ & a : C_1, \quad b : C,\ (b, c) : r,\ c : (A \sqcap B) \,\} & \\
\mathcal{D} = \{ & \dfrac{x : C_1 \mid x : \neg D}{x : \neg D}, \quad \dfrac{x : C_2 \mid x : D}{x : D} \;\} &
\end{array}
$$